Projects in Wireless Communication
Carrier Transmission

Fredrik Tufvesson

Department of Electrical and Information Technology

Lund University, Sweden

Lund, Fall 2024

Carrier Transmission

The transmitted signal is $y(t) = \sum_k a_k h(t - kT)$.

What is the bandwidth?
More generally, what is its Fourier transform?

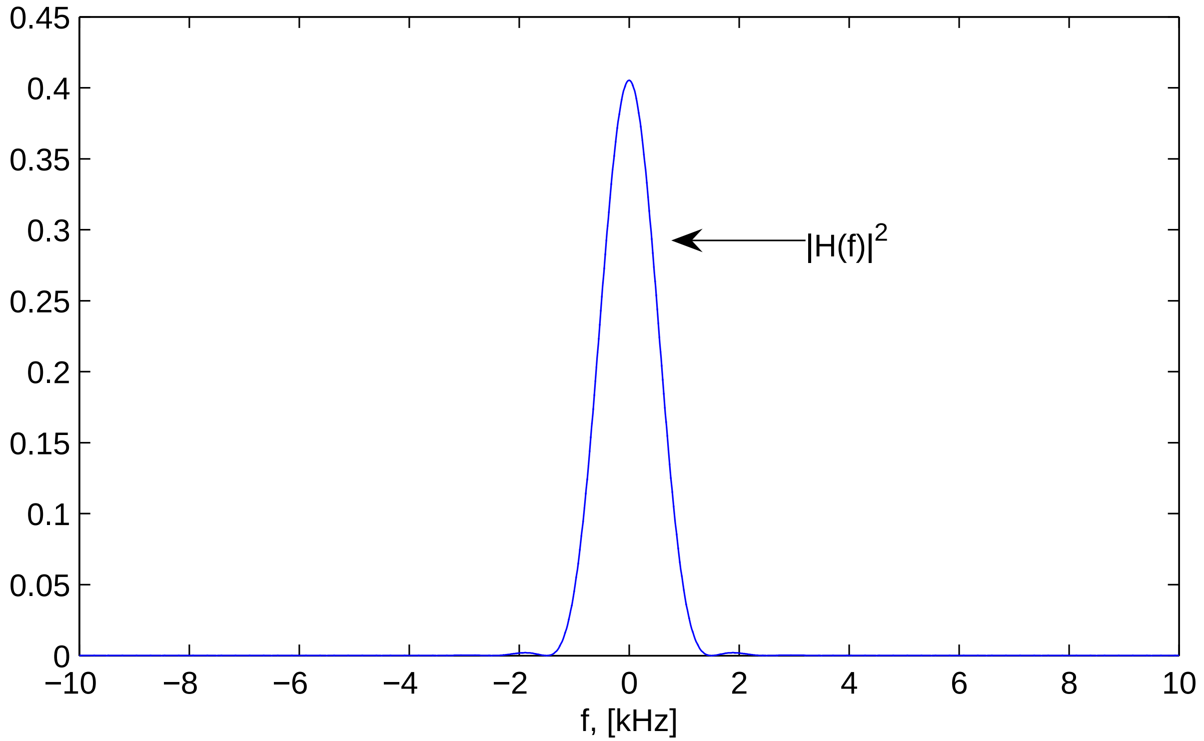
Carrier Transmission

1.	Linearity	$ax_1(t) + bx_2(t) \leftrightarrow aX_1(f) + bX_2(f)$
2.	Inverse	$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j\omega t} df$
3.	Translation (time shift)	$x(t - t_0) \leftrightarrow X(f)\, e^{-j\omega t_0}$
4.	Modulation (frequency shift)	$x(t)\, e^{j\omega_0 t} \leftrightarrow X(f - f_0)$
		$x(t) \cos \omega_0 t \leftrightarrow \frac{1}{2} X(f + f_0) + \frac{1}{2} X(f - f_0)$
5.	Time scaling	$x(at) \leftrightarrow \frac{1}{\lvert a \rvert} X(f/a)$
6.	Differentiation in time	$\frac{d}{dt} x(t) \leftrightarrow j\omega X(f)$
7.	Integration in time	$\int_{-\infty}^{t} x(\tau) d\tau \leftrightarrow \frac{1}{j\omega} X(f)$
8.	Duality	$X(t) \leftrightarrow x(-f)$
9.	Conjugate functions	$x^*(t) \leftrightarrow X^*(-f)$
10.	Convolution in time	$x_1(t) * x_2(t) \leftrightarrow X_1(f) X_2(f)$
11.	Multiplication in time	$x_1(t) x_2(t) \leftrightarrow X_1(f) * X_2(f)$
12.	Parseval's formulas	$\int_{-\infty}^{\infty} x_1(t) x_2^*(t) dt = \int_{-\infty}^{\infty} X_1(f) X_2^*(f) df$
		or, when $x_1(t) = x_2(t)$,
		$\int_{-\infty}^{\infty} \lvert x(t) \rvert^2 dt = \int_{-\infty}^{\infty} \lvert X(f) \rvert^2 df$

Table 1: Properties of the Fourier transform.

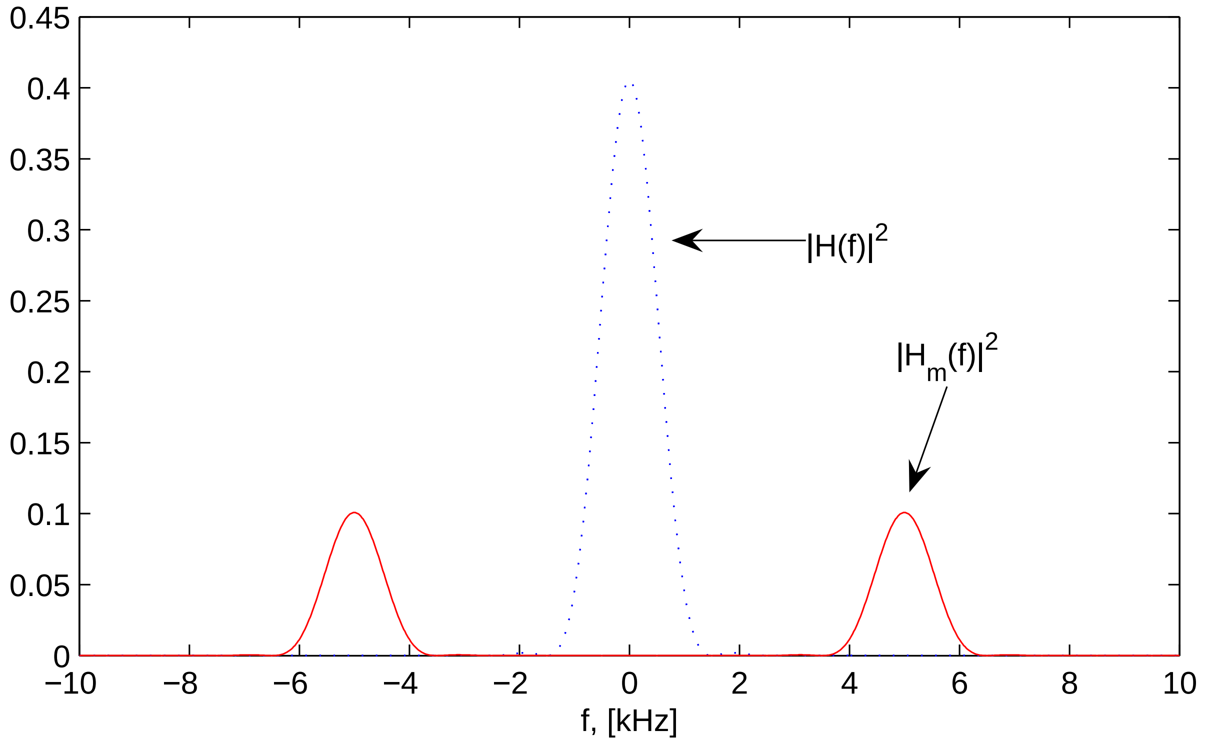
Carrier Transmission

The baseband signal is $y(t) = \sum_k a_k h(t - kT)$. The power spectral density of the transmission is $\propto |H(f)|^2$



Carrier Transmission

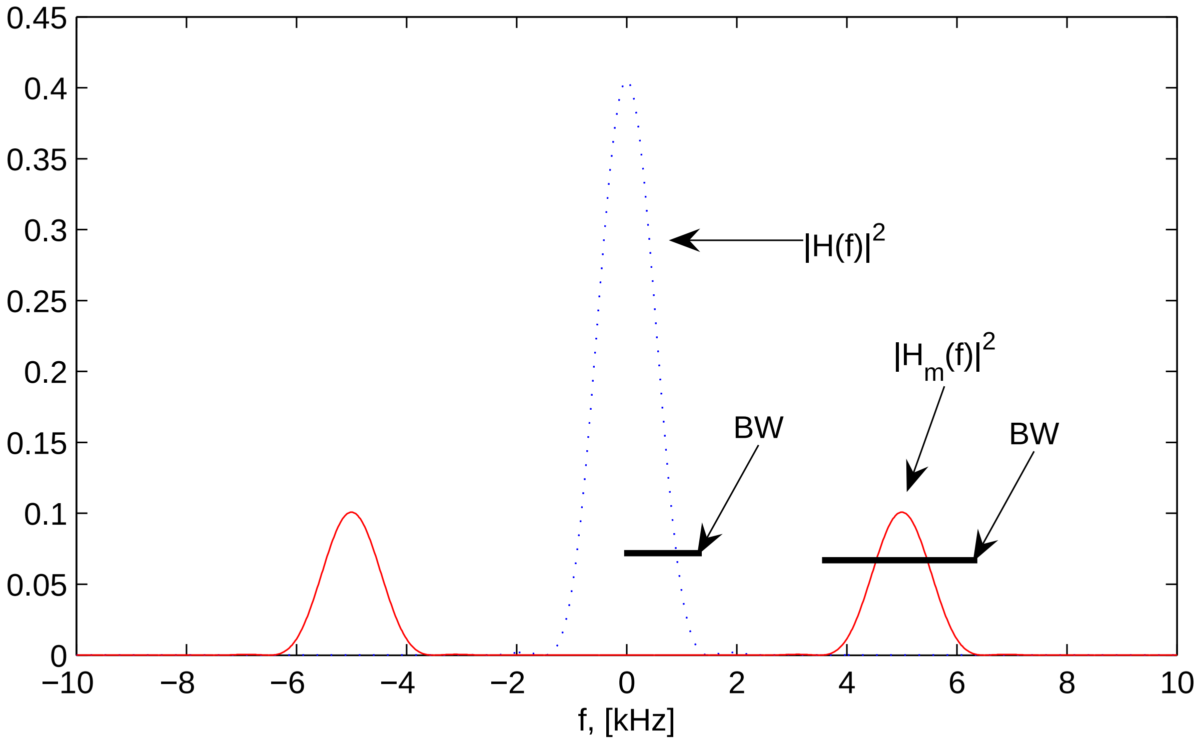
The baseband signal is $y(t) = \sum_k a_k h(t - kT)$. The power spectral density of the transmission is $\propto |H(f)|^2$



The carrier modulated signal is $y_m(t) = y(t)\cos(2\pi t f_c)$

Carrier Transmission

The baseband signal is $y(t) = \sum_k a_k h(t - kT)$. The power spectral density of the transmission is $\propto |H(f)|^2$



The carrier modulated signal is $y_m(t) = y(t)\cos(2\pi t f_c)$

But bandwidth gets twice as large!

Carrier Transmission

Where did the energy go?

Basic Fourier relations:

$$\cos(2\pi f_c t)h(t) \longleftrightarrow \frac{1}{2}H(f-f_c)+\frac{1}{2}H(f+f_c)$$

$$\sin(2\pi f_c t)h(t) \longleftrightarrow \frac{i}{2}H(f-f_c)-\frac{i}{2}H(f+f_c)$$

Carrier Transmission

Where did the energy go?

Basic Fourier relations:

$$\cos(2\pi f_c t)h(t) \longleftrightarrow \frac{1}{2}H(f-f_c)+\frac{1}{2}H(f+f_c)$$

$$\sin(2\pi f_c t)h(t) \longleftrightarrow \frac{i}{2}H(f-f_c)-\frac{i}{2}H(f+f_c)$$

The $1/2$ factor corresponds to a $1/4$ of the energy. Since there are two terms, $1/2$ of the energy is preserved.

Carrier Transmission

Where did the energy go?

Basic Fourier relations:

$$\cos(2\pi f_c t)h(t) \longleftrightarrow \frac{1}{2}H(f - f_c) + \frac{1}{2}H(f + f_c)$$

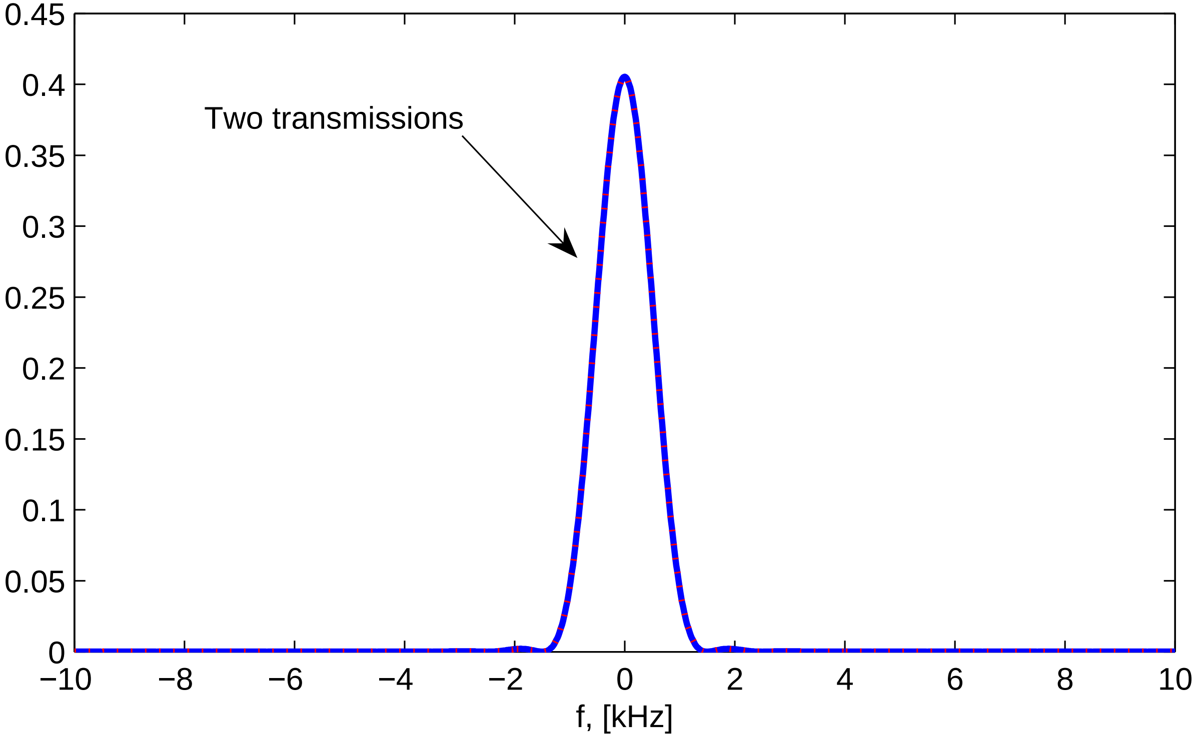
$$\sin(2\pi f_c t)h(t) \longleftrightarrow \frac{i}{2}H(f - f_c) - \frac{i}{2}H(f + f_c)$$

The $1/2$ factor corresponds to a $1/4$ of the energy. Since there are two terms, $1/2$ of the energy is preserved.

What about the increased bandwidth?

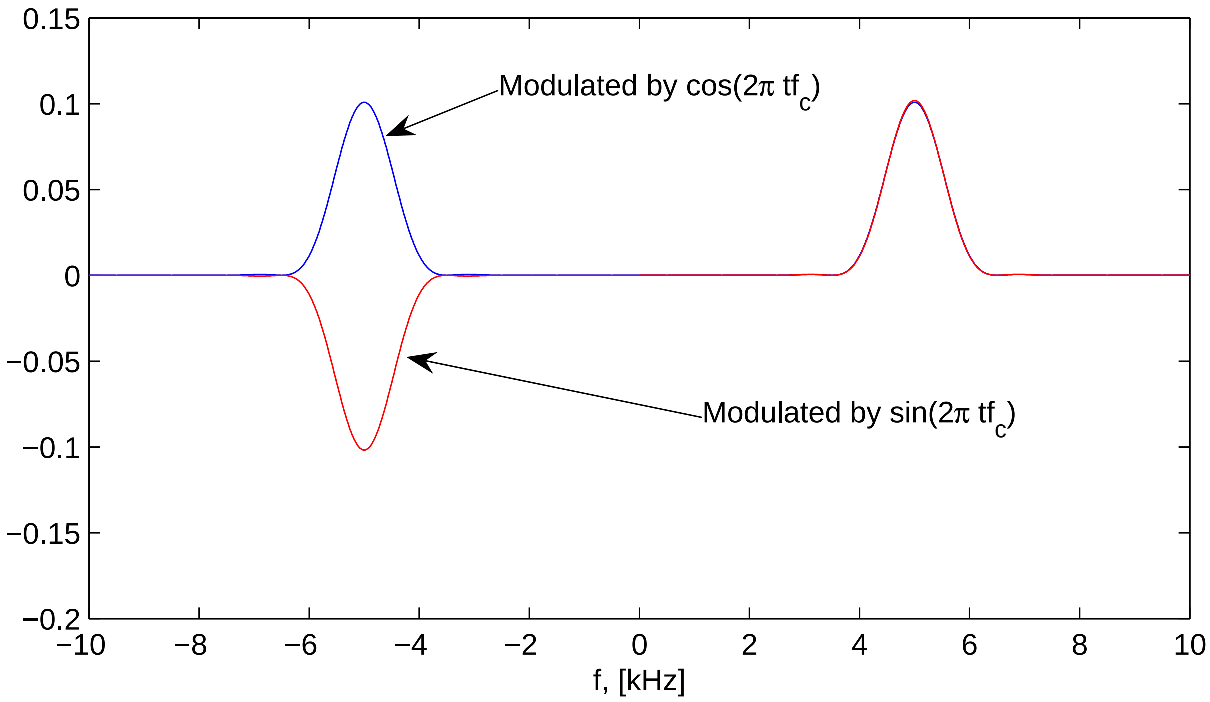
Carrier Transmission

Assume two independent baseband transmissions



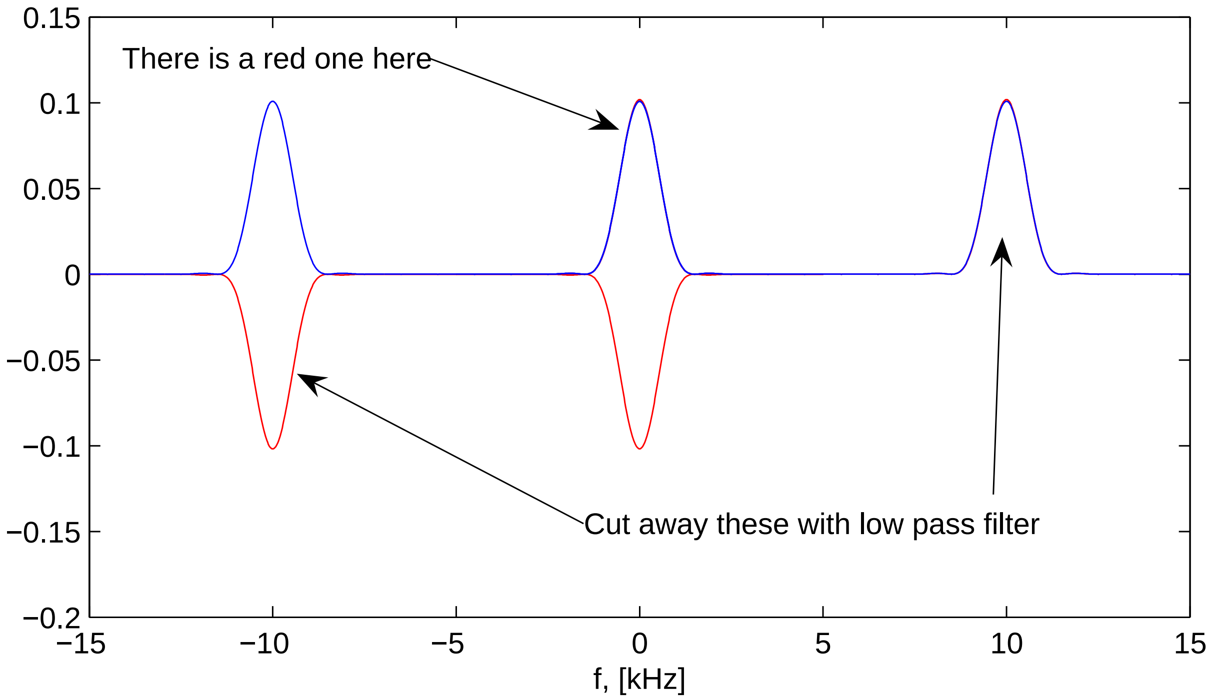
Carrier Transmission

Assume two independent baseband transmissions
After modulation with $\cos(2\pi t f_c)$ and $\sin(2\pi t f_c)$ we get



Carrier Transmission

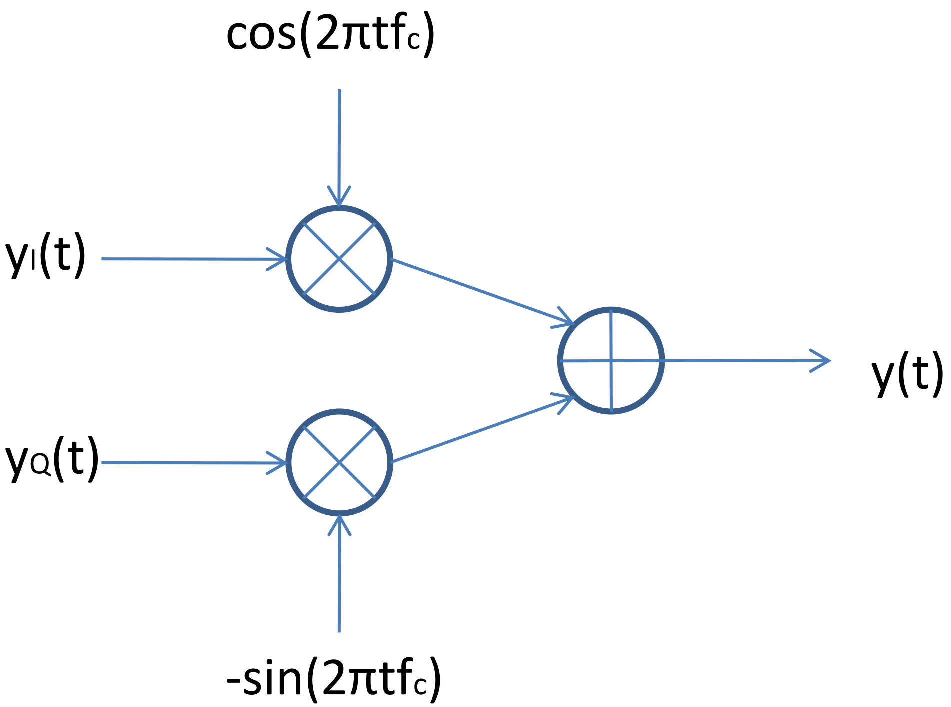
Assume two independent baseband transmissions
After demodulation with $\cos(2\pi t f_c)$ we get



The red spectras around f=0 cancel out, thus, we can detect the blue independently from the red. Equivalent for demodulation with $\sin(2\pi t f_c)$

Carrier Transmission

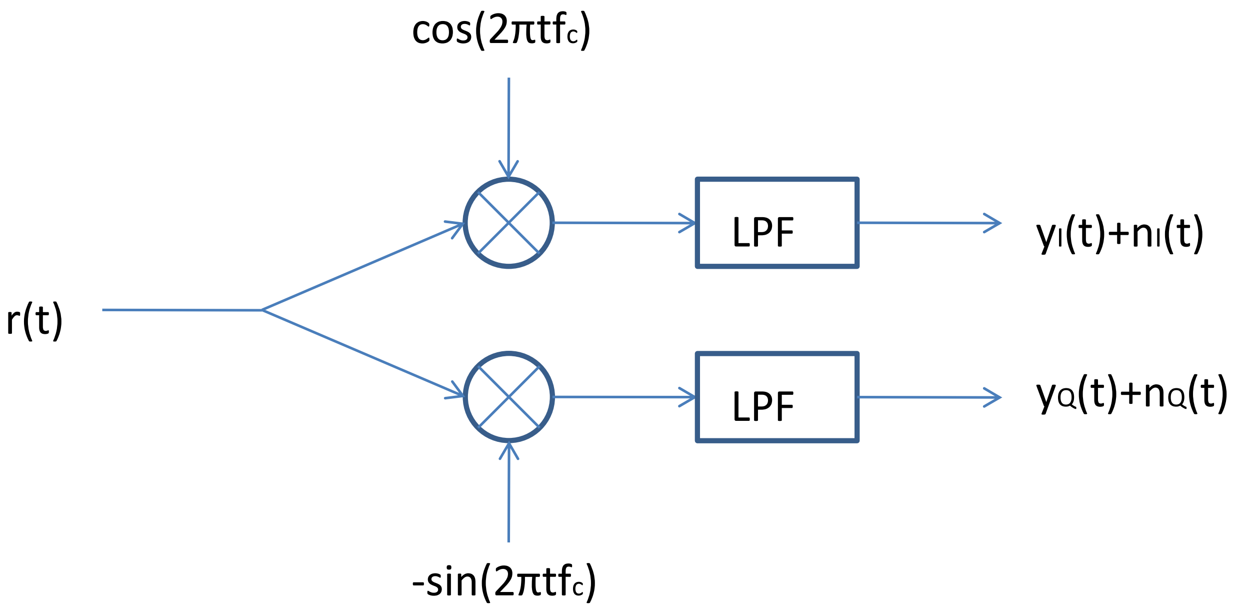
The block diagram of the transmitter is



$$y(t) = y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t)$$

Carrier Transmission

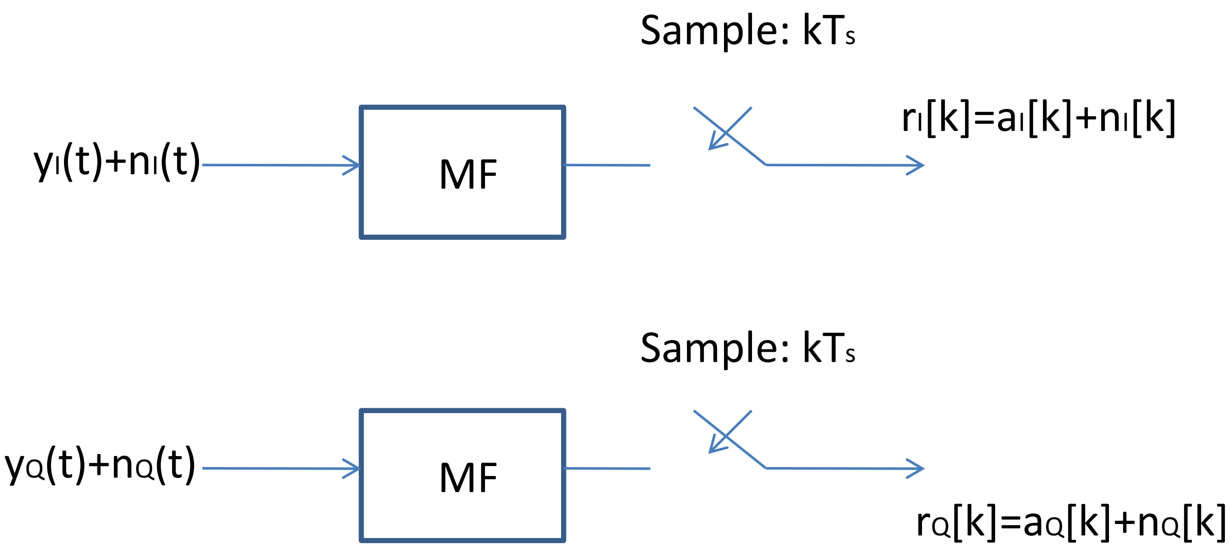
The block diagram of the receiver is



The in-phase and the quadrature components can be independently detected!
The LPF (low pass filters) can be taken as a matched filter to $h(t)$

Carrier Transmission

The signals at both rails are baseband signals, and conventional processing follows: matched filter $\rightarrow$ sampling every T_s second $\rightarrow$ decision unit



What is a complex-valued symbol $1+i$?

In QPSK, we transmit complex valued symbols. In one symbol interval, we have

$$y(t) = \underbrace{h(t)}_{y_I(t)} \cos(2\pi f_c t) - \underbrace{h(t)}_{y_Q(t)} \sin(2\pi f_c t)$$

What is a complex-valued symbol $1+i$?

In QPSK, we transmit complex valued symbols. In one symbol interval, we have

$$y(t) = \underbrace{h(t)}_{y_I(t)} \cos(2\pi f_c t) - \underbrace{h(t)}_{y_Q(t)} \sin(2\pi f_c t)$$

Carrier Transmission

We can alternatively express the signal $y(t)$ as

$$\begin{aligned} y(t) &= y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t) \\ &= e(t)\cos(2\pi f_c t + \theta(t)) \end{aligned}$$

where $e(t)$ is the envelope and $\theta(t)$ is the phase

For QPSK, $e(t) = \sqrt{2}h(t)$ and $\theta(t) \in \{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$

We can further manipulate $y(t)$ into

$$\begin{aligned} y(t) &= \mathrm{Re}\{(y_I(t) + iy_Q(t))\mathrm{e}^{i2\pi f_c t}\} \\ &= \mathrm{Re}\{\tilde{y}(t)\mathrm{e}^{i2\pi f_c t}\} \end{aligned}$$

where

$$\tilde{y}(t) = y_I(t) + iy_Q(t)$$

Example

Assume that we have two bits to transmit, say +1 and -1.

Carrier Transmission

Example

Assume that we have two bits to transmit, say +1 and -1.

We can either do this as

$$y(t) = h(t)\cos(2\pi f_c t) - (-h(t))\sin(2\pi f_c t)$$

Example

Assume that we have two bits to transmit, say +1 and -1.

We can either do this as

$$y(t) = h(t)\cos(2\pi f_c t) - (-h(t))\sin(2\pi f_c t)$$

or as

$$y(t) = \sqrt{2}h(t)\cos(2\pi f_c t + 7\pi/4)$$

Example

Assume that we have two bits to transmit, say +1 and -1.

We can either do this as

$$y(t) = h(t)\cos(2\pi f_c t) - (-h(t))\sin(2\pi f_c t)$$

or as

$$y(t) = \sqrt{2}h(t)\cos(2\pi f_c t + 7\pi/4)$$

or as

$$y(t) = \mathrm{Re}\{(1-i)h(t)\mathrm{e}^{i2\pi f_c t}\}$$

Carrier Transmission

In the last representation, we can change the receiver processing into