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Project in Wireless Communication Lecture 2, IQ modulation

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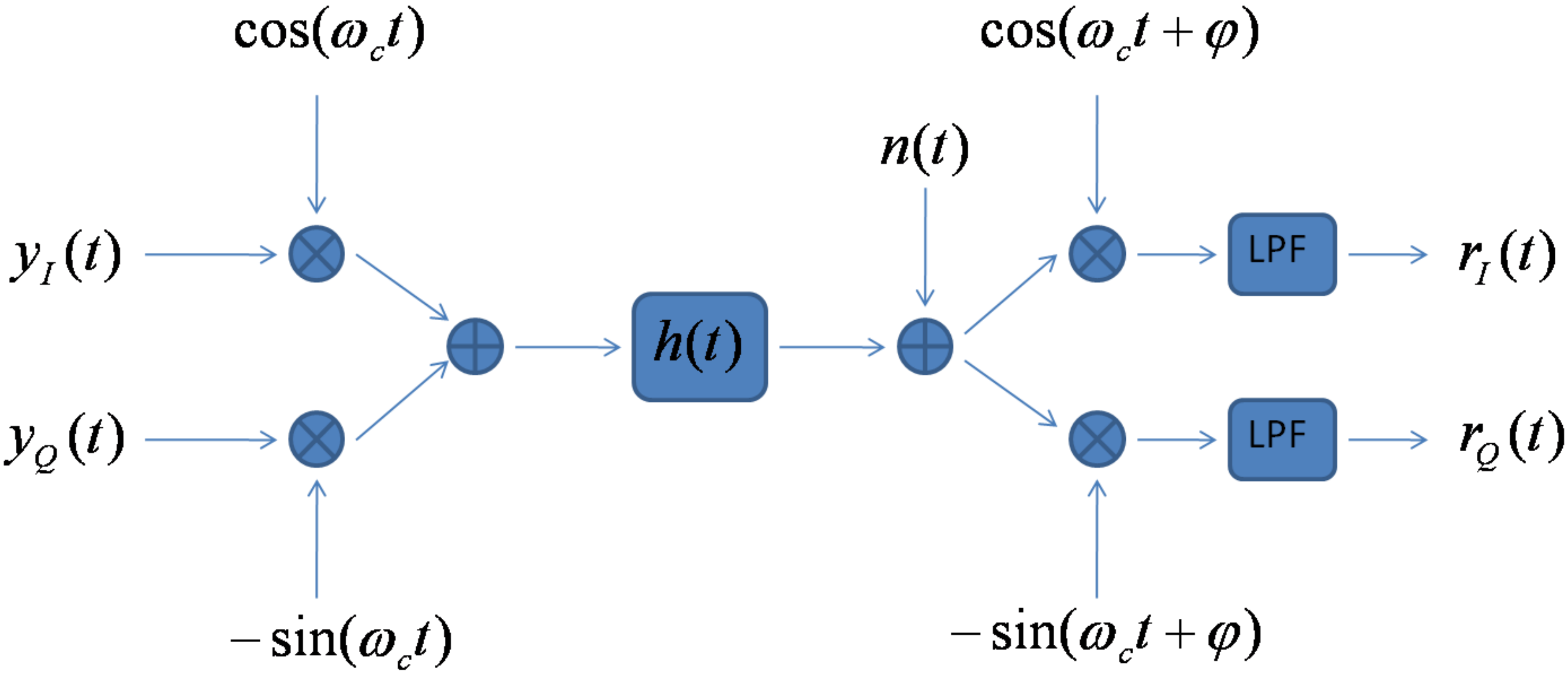
ELECTRICAL AND INFORMATION TECHNOLOGY

System model



We want to represent the outputs as functions of the inputs
Note that the receiver and transmitter are not synchronous

Models of input and channel

The transmitted signal $y(t)$ equals

$$y(t) = y_I(t)\cos(\omega_c t) - y_Q(t)\sin(\omega_c t).$$

Similarly, the channel impulse response can be expressed as

$$h(t) = h_I(t)\cos(\omega_c t) - h_Q(t)\sin(\omega_c t).$$

Channel output in the Fourier domain

To evaluate $r(t) = y(t) * h(t)$, we consider the signals in the Fourier domain:

$$\begin{aligned} R(f) &= Y(f)H(f) \\ &= \frac{1}{4}\left[Y_I(f+f_c) + Y_I(f-f_c) + \jmath Y_Q(f+f_c) - \jmath Y_Q(f-f_c)\right] \\ &\quad \times \left[H_I(f+f_c) + H_I(f-f_c) + \jmath H_Q(f+f_c) - \jmath H_Q(f-f_c)\right] \end{aligned}$$

Now observe that a product of the type $Y_{I/Q}(f \pm f_c) H_{I/Q}(f \mp f_c) = 0$

Channel output in the Fourier domain

$$
\begin{aligned}
R(f) \quad = \quad & \frac{1}{4}\left[Y_I(f+f_c)H_I(f+f_c) + \jmath Y_I(f+f_c)H_Q(f+f_c) + Y_I(f-f_c)H_I(f-f_c)\right. \\
& -\jmath Y_I(f-f_c)H_Q(f-f_c) + \jmath Y_Q(f+f_c)H_I(f+f_c) - Y_Q(f+f_c)H_Q(f+f_c) \\
& \left.-\jmath Y_Q(f-f_c)H_I(f+f_c) - Y_Q(f-f_c)H_Q(f-f_c)\right]
\end{aligned}
$$

Channel output in the Fourier domain

$$
\begin{aligned}
R(f) \;=\; & \frac{1}{4}\big[Y_I(f+f_c)H_I(f+f_c) + \jmath Y_I(f+f_c)H_Q(f+f_c) + Y_I(f-f_c)H_I(f-f_c) \\
& -\jmath Y_I(f-f_c)H_Q(f-f_c) + \jmath Y_Q(f+f_c)H_I(f+f_c) - Y_Q(f+f_c)H_Q(f+f_c) \\
& -\jmath Y_Q(f-f_c)H_I(f+f_c) - Y_Q(f-f_c)H_Q(f-f_c)\big]
\end{aligned}
$$

By identifying terms, we get that

$$r(t) = \tilde{r}_I(t)\cos(\omega_c t) - \tilde{r}_Q(t)\sin(\omega_c t),$$

with

$$\tilde{r}_I(t) = \frac{1}{2}[y_I(t) * h_I(t) - y_Q(t) * h_Q(t)]$$

and

$$\tilde{r}_Q(t) = \frac{1}{2}[y_I(t) * h_Q(t) + y_Q(t) * h_I(t)].$$

Channel output in the Fourier domain

$$R(f) = \frac{1}{4}[Y_I(f+f_c)H_I(f+f_c) + \jmath Y_I(f+f_c)H_Q(f+f_c) + Y_I(f-f_c)H_I(f-f_c)$$
$$-\jmath Y_I(f-f_c)H_Q(f-f_c) + \jmath Y_Q(f+f_c)H_I(f+f_c) - Y_Q(f+f_c)H_Q(f+f_c)$$
$$-\jmath Y_Q(f-f_c)H_I(f+f_c) - Y_Q(f-f_c)H_Q(f-f_c)]$$

By identifying terms, we get that

$$r(t) = \tilde{r}_I(t)\cos(\omega_c t) - \tilde{r}_Q(t)\sin(\omega_c t),$$

with

$$\tilde{r}_I(t) = \frac{1}{2}[y_I(t) * h_I(t) - y_Q(t) * h_Q(t)]$$

and

$$\tilde{r}_Q(t) = \frac{1}{2}[y_I(t) * h_Q(t) + y_Q(t) * h_I(t)].$$

Channel output in the Fourier domain

$$
\begin{aligned}
R(f) \;=\; & \frac{1}{4}\big[Y_I(f+f_c)H_I(f+f_c) + \jmath Y_I(f+f_c)H_Q(f+f_c) + Y_I(f-f_c)H_I(f-f_c) \\
& -\jmath Y_I(f-f_c)H_Q(f-f_c) + \jmath Y_Q(f+f_c)H_I(f+f_c) - Y_Q(f+f_c)H_Q(f+f_c) \\
& -\jmath Y_Q(f-f_c)H_I(f+f_c) - Y_Q(f-f_c)H_Q(f-f_c)\big]
\end{aligned}
$$

By identifying terms, we get that

$$r(t) = \tilde{r}_I(t)\cos(\omega_c t) - \tilde{r}_Q(t)\sin(\omega_c t),$$

with

$$\tilde{r}_I(t) = \frac{1}{2}[y_I(t) * h_I(t) - y_Q(t) * h_Q(t)]$$

and

$$\tilde{r}_Q(t) = \frac{1}{2}[y_I(t) * h_Q(t) + y_Q(t) * h_I(t)].$$

Channel output in the Fourier domain

$$\begin{aligned} R(f) &= \frac{1}{4}[Y_I(f+f_c)H_I(f+f_c) + \jmath Y_I(f+f_c)H_Q(f+f_c) + Y_I(f-f_c)H_I(f-f_c) \\ &\quad -\jmath Y_I(f-f_c)H_Q(f-f_c) + \jmath Y_Q(f+f_c)H_I(f+f_c) - Y_Q(f+f_c)H_Q(f+f_c) \\ &\quad -\jmath Y_Q(f-f_c)H_I(f+f_c) - Y_Q(f-f_c)H_Q(f-f_c)] \end{aligned}$$

By identifying terms, we get that

$$r(t) = \tilde{r}_I(t)\cos(\omega_c t) - \tilde{r}_Q(t)\sin(\omega_c t),$$

with

$$\tilde{r}_I(t) = \frac{1}{2}[y_I(t) * h_I(t) - y_Q(t) * h_Q(t)]$$

and

$$\tilde{r}_Q(t) = \frac{1}{2}[y_I(t) * h_Q(t) + y_Q(t) * h_I(t)].$$

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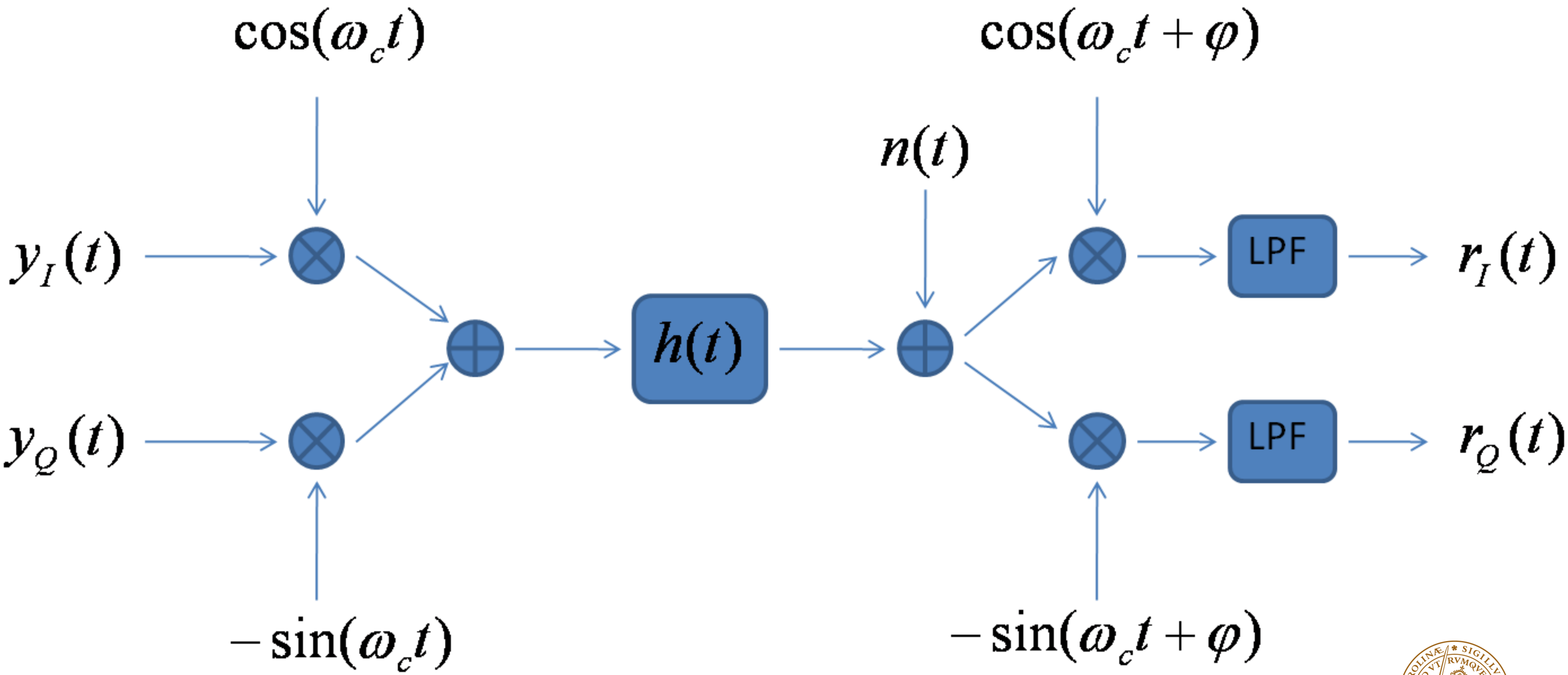
Channel output in the time domain



Now, lets multiply by $\cos(\omega_c t + \varphi)$

Channel output in the time domain

Basic trigonometric properties

$$\begin{aligned}\cos(x)\cos(y) &= \tfrac{1}{2}[\cos(x+y)+\cos(x-y)]\\ \sin(x)\cos(y) &= \tfrac{1}{2}[\sin(x+y)+\sin(x-y)]\\ \sin(x)\sin(y) &= \tfrac{1}{2}[\cos(x+y)-\cos(x-y)]\end{aligned}$$

Signal at upper rail equals

$$[\tilde{r}_I(t)\cos(\omega_c t)-\tilde{r}_Q(t)\sin(\omega_c t)]\cos(\omega_c t+\phi)=$$

$$\frac{1}{2}\Big[\,\tilde{r}_I(t)[\cos(2\omega_c t+\phi)+\cos(\phi)]-\,\tilde{r}_Q(t)[\sin(2\omega_c t+\phi)-\sin(\phi)]\Big]$$

Remove by low pass filtering

Channel output in the time domain

We get that

$$
\begin{aligned}
r_I(t) &= \frac{1}{2}\left[\tilde{r}_I(t)\cos(\phi) + \tilde{r}_Q(t)\sin(\phi)\right] \\
&= \frac{1}{4}\left[(y_I(t) * h_I(t) - y_Q(t) * h_Q(t))\cos(\phi) + (y_I(t) * h_Q(t) + y_Q(t) * h_I(t)))\sin(\phi)\right]
\end{aligned}
$$

and

$$
r_Q(t) = \frac{1}{4}\left[-(y_I(t) * h_I(t) - y_Q(t) * h_Q(t))\sin(\phi) + (y_I(t) * h_Q(t) + y_Q(t) * h_I(t))\cos(\phi)\right]
$$

Channel output in the time domain

Now construct the two complex valued signals

$$y^c(t) \triangleq y_I(t) + \jmath y_Q(t)$$

and

$$r^c(t) \triangleq r_I(t) + \jmath r_Q(t).$$

By identifying some terms we can conclude that

$$r^c(t) = y^c(t) * h^c(t),$$

with $h^c(t) \triangleq h_I(t)\cos(\phi) + h_Q(t)\sin(\phi) + \jmath(h_Q(t)\cos(\phi) - h_I(t)\sin(\phi)).$

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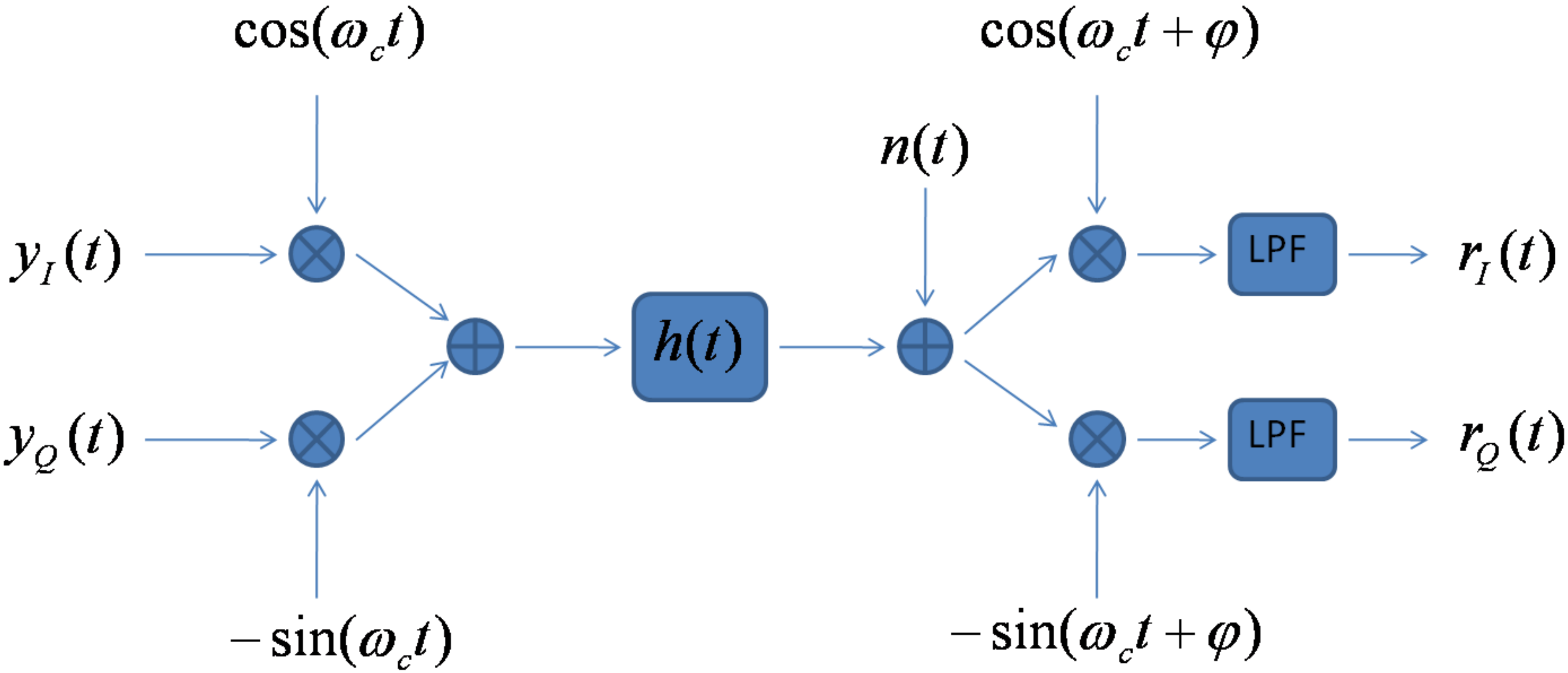
Final result



This can be modeled in the complex baseband as

Final result

$$h^c(t) \triangleq h_I(t)\cos(\phi) + h_Q(t)\sin(\phi) + \jmath(h_Q(t)\cos(\phi) - h_I(t)\sin(\phi)).$$

$$y^c(t) = y_I(t) + \jmath y_Q(t)$$

$$r^c(t) = r_I(t) + \jmath r_Q(t)$$

$$r^c(t) = y^c(t) \star h^c(t)$$

What is the effect of ϕ ?

$$h^c(t) \triangleq h_I(t)\cos(\phi) + h_Q(t)\sin(\phi) + \jmath(h_Q(t)\cos(\phi) - h_I(t)\sin(\phi)).$$

Energy of the impulse response

$$\int_{-\infty}^{\infty} |h^c(t)|^2 dt = \int_{-\infty}^{\infty} h_I^2(t) + h_Q^2(t) dt$$

The energy is independent of ϕ!

It doesn't matter if Tx and Rx are not synchronous

Conclusion

We can always work in the complex baseband domain with the input/output relation

$$r^c(t) = y^c(t) \star h^c(t) + n^c(t)$$

And we do not care about ϕ (it must be estimated though)

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