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Project in Wireless Communication
Lecture 3, Synchronization

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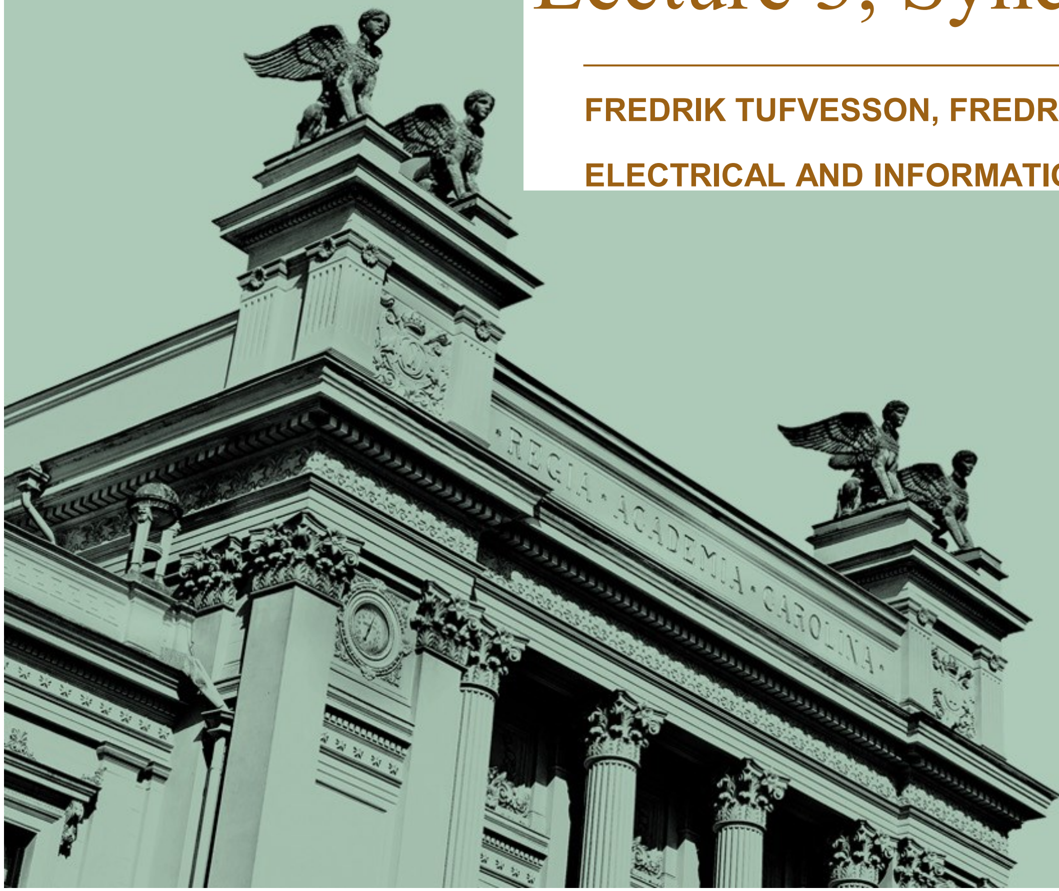
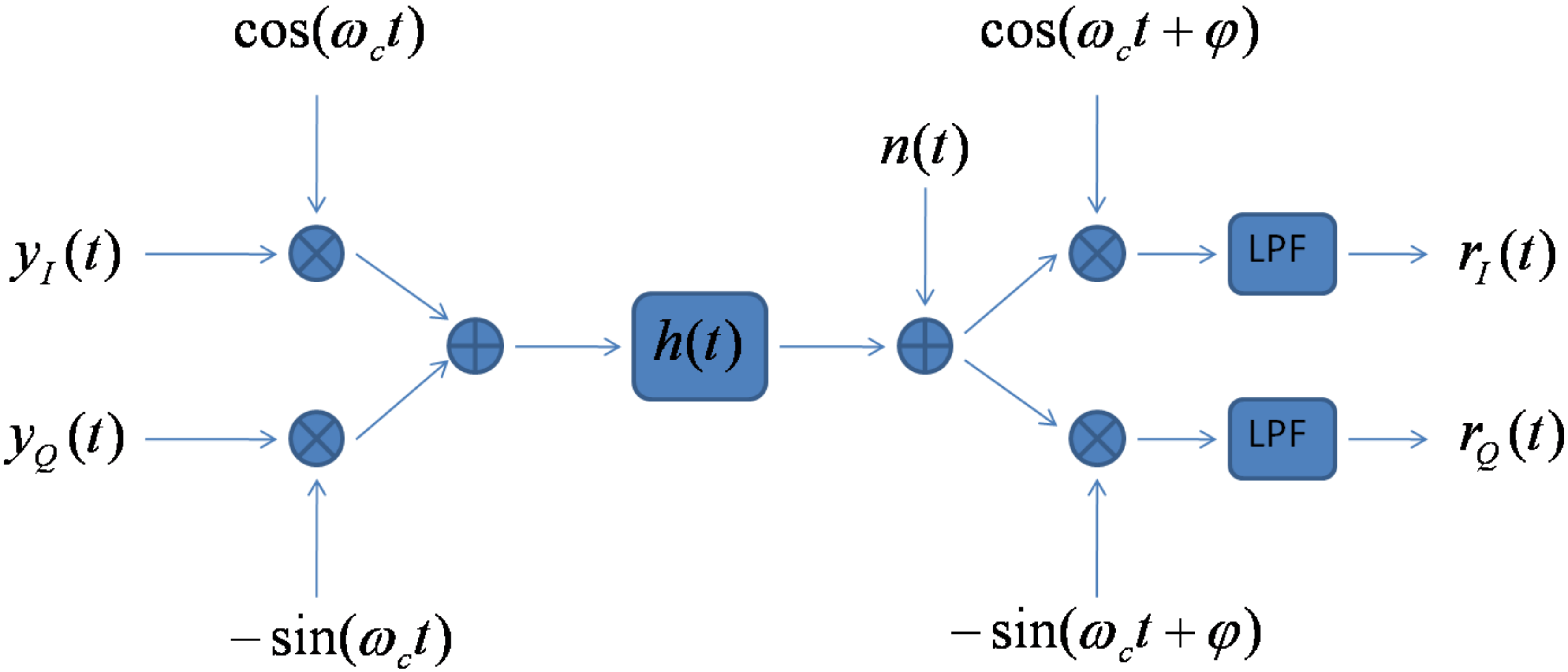
ELECTRICAL AND INFORMATION TECHNOLOGY



Final result, complex representation



This can be modeled in the complex baseband as

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Final result, complex representation

$$h^c(t) \triangleq h_I(t)\cos(\phi) + h_Q(t)\sin(\phi) + \jmath(h_Q(t)\cos(\phi) - h_I(t)\sin(\phi)).$$

$$y^c(t) = y_I(t) + \jmath y_Q(t)$$

$$r^c(t) = r_I(t) + \jmath r_Q(t)$$

$$r^c(t) = y^c(t) \star h^c(t)$$

Conclusion

We can always work in the complex baseband domain with the input/output relation

$$r^c(t) = y^c(t) \star h^c(t) + n^c(t)$$

where

$$y^c(t) = \sum_k a_k p(t - kT_s)$$

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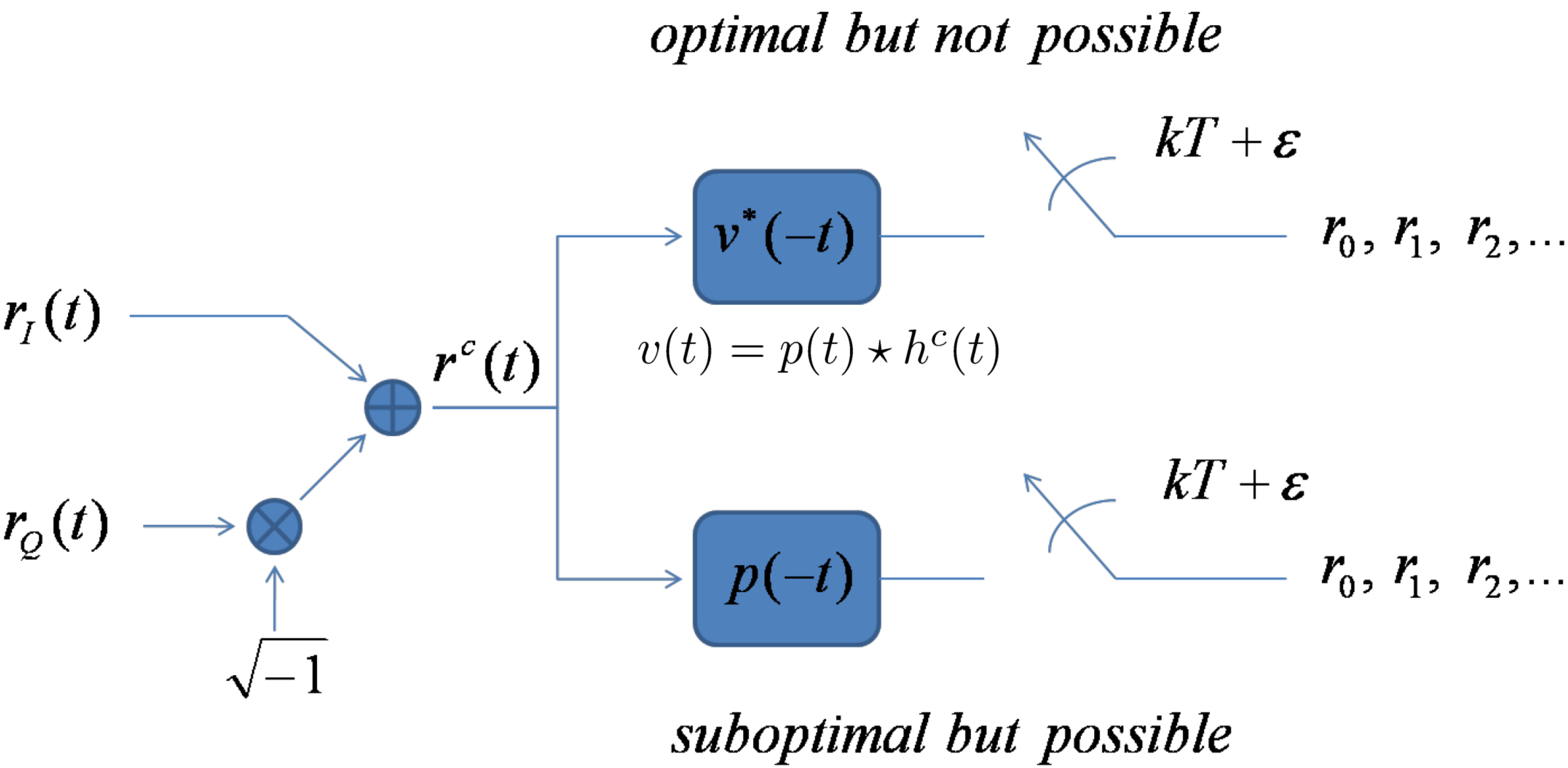
Matched filtering



Matched filter output

Let $z(t)$ denote the matched filter. The signal part of each sample r_k equals

$$\begin{aligned}
r_k &= \left.\int_{-\infty}^{\infty} r^c(\tau) z(t-\tau) \mathrm{d}\tau \right|_{t=kT+\epsilon} \\
&= \left.\int_{-\infty}^{\infty} r^c(\tau) p(\tau - t) \mathrm{d}\tau \right|_{t=kT+\epsilon} \\
&= \int_{-\infty}^{\infty} \left(\sum_{\ell} a_\ell v(\tau - \ell T) \right) p(\tau - kT - \epsilon) \mathrm{d}\tau \\
&= \sum_{\ell} a_\ell \int_{-\infty}^{\infty} v(\tau - \ell T) p(\tau - kT - \epsilon) \mathrm{d}\tau \\
&= \sum_{\ell} a_\ell \int_{-\infty}^{\infty} v(\tau) p(\tau + (\ell - k)T - \epsilon) \mathrm{d}\tau \\
&= \sum_{\ell} a_\ell h_{k-\ell},
\end{aligned}$$

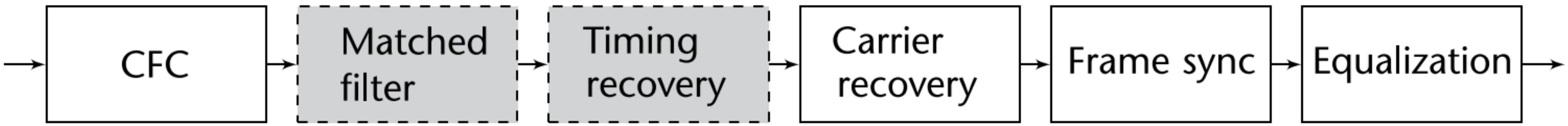
where

$$h_n \triangleq \int_{-\infty}^{\infty} v(\tau) p(\tau - nT - \epsilon) \mathrm{d}\tau.$$

Examples

In ”Software Defined Radio for Engineers”, by Collins et al, different aspects of synchronization is dealt with in chapter 6-9.

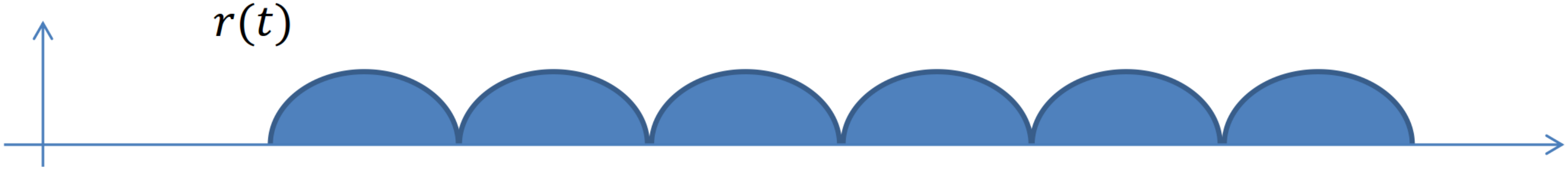
Let’s now consider timing recovery after the matched filter.



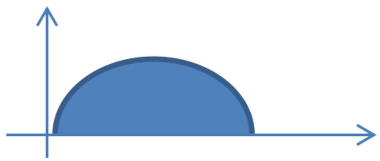
Source: Software Defined Radio for Engineers, Collins et al

Synch in the project

$r(t)$

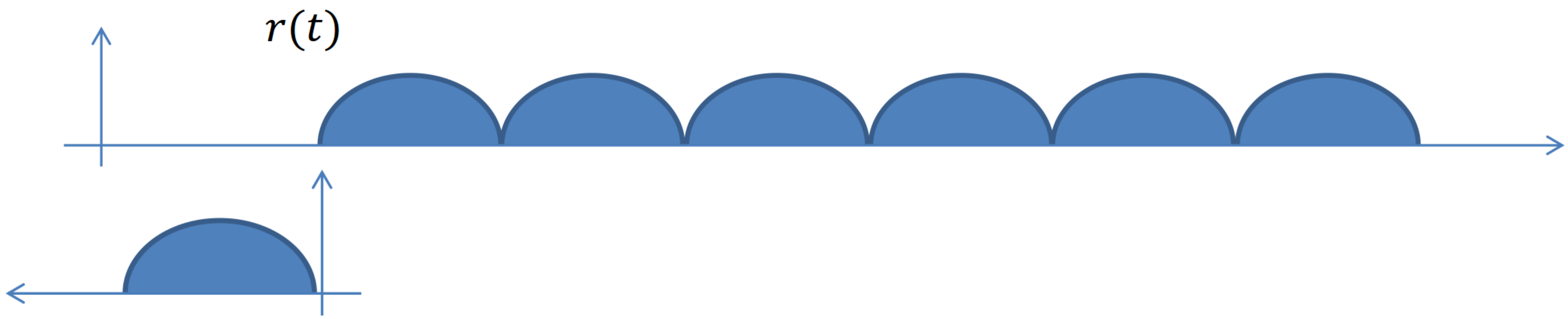


MF $\quad$ p(t)



Operation of the MF: $\int p(\tau)r(t-\tau)d\tau$

Synch in the project



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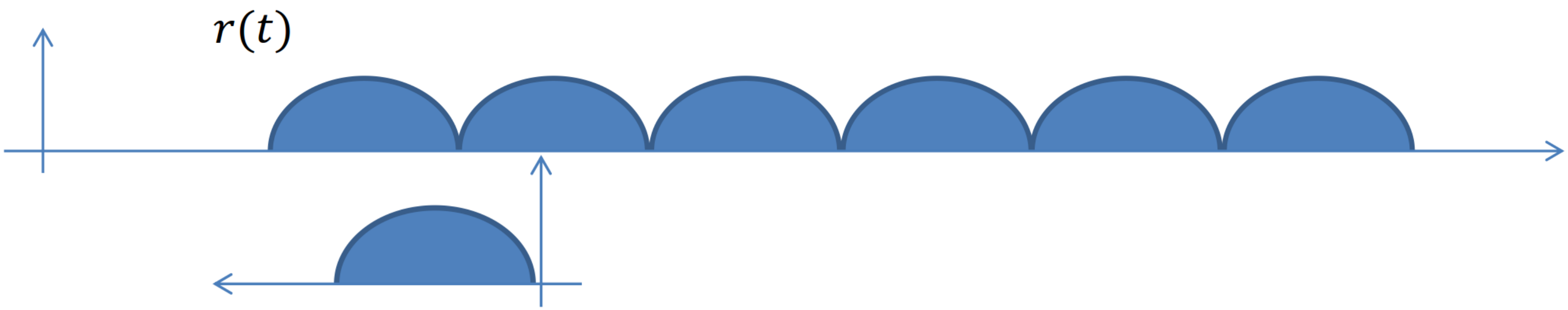

Operation of the MF: $\int p(\tau)r(t-\tau)d\tau$

Graphical interpretation:
1. Let *$p(-\tau)$* slide along the *x*-axis
2. At each position, multiply *$p(-\tau)$* and *$r(\tau)$*
3. Integrate the product

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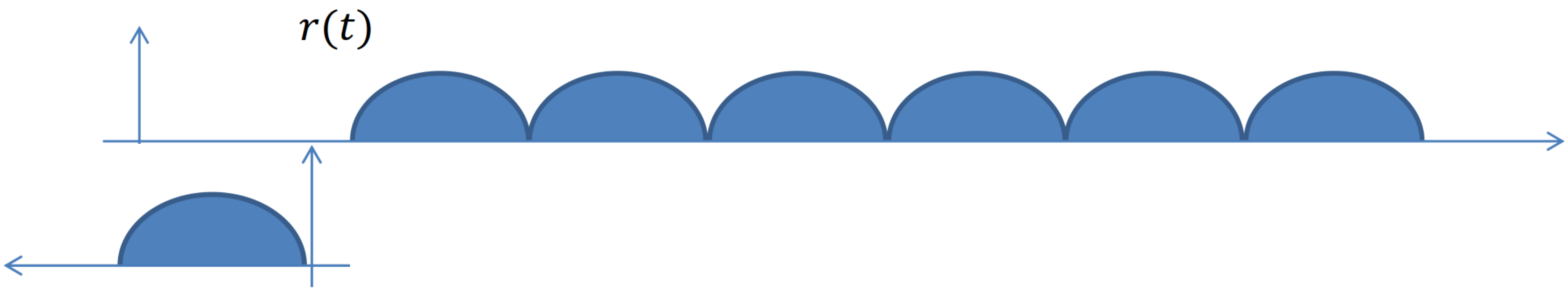
Synch in the project

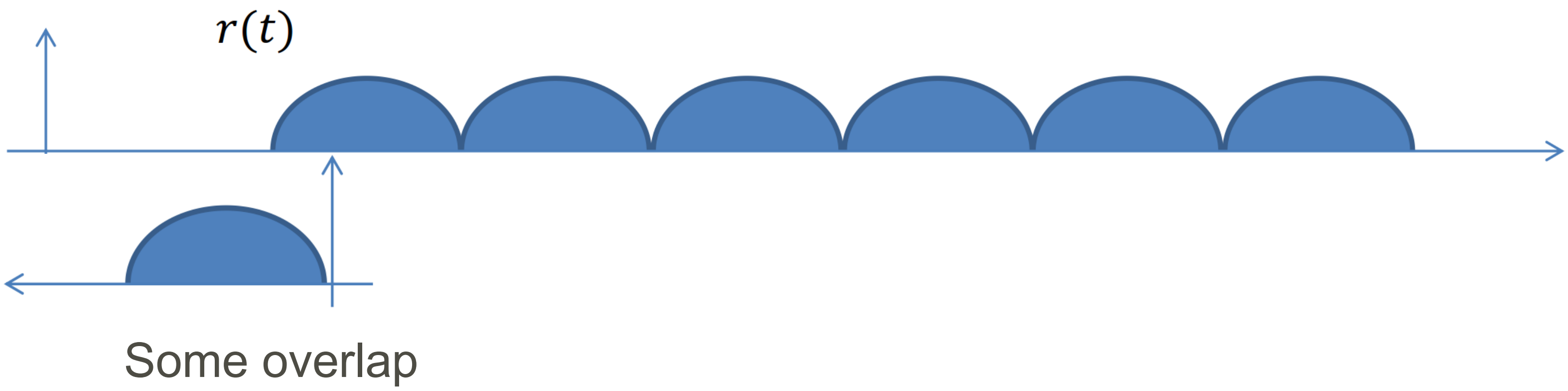


Only noise

Operation of the MF: $\int p(\tau)r(t-\tau)d\tau$

Graphical interpretation:
1. Let *p(-τ)* slide along the *x*-axis
2. At each position, multiply *p(-τ)* and *r(τ)*
3. Integrate the product

Synch in the project



Operation of the MF: $\int p(\tau)r(t-\tau)d\tau$

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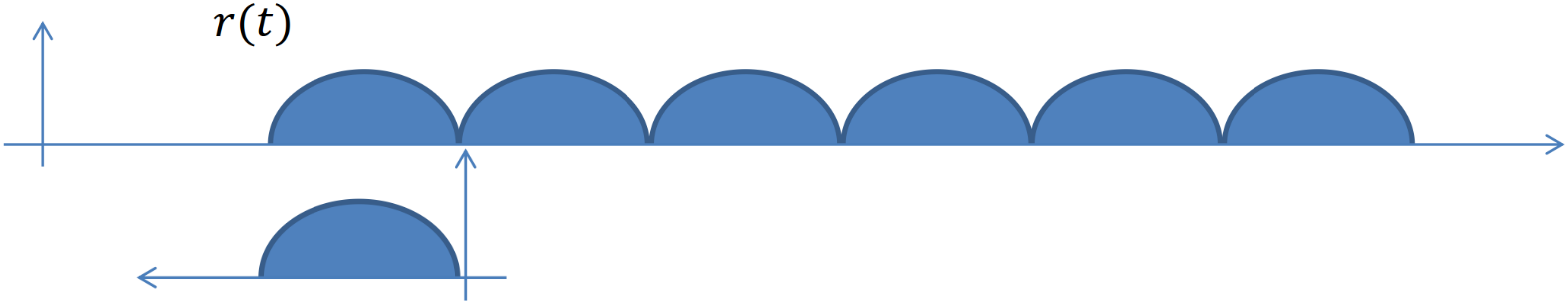
Synch in the project



Full overlap, max value

Operation of the MF: $\int p(\tau)r(t-\tau)d\tau$

Graphical interpretation:
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Recall Cauchy-Schwarz: $|\int f(x)g(x)dx| \leq \sqrt{\int (f(x))^2dx}\sqrt{\int (g(x))^2dx}$

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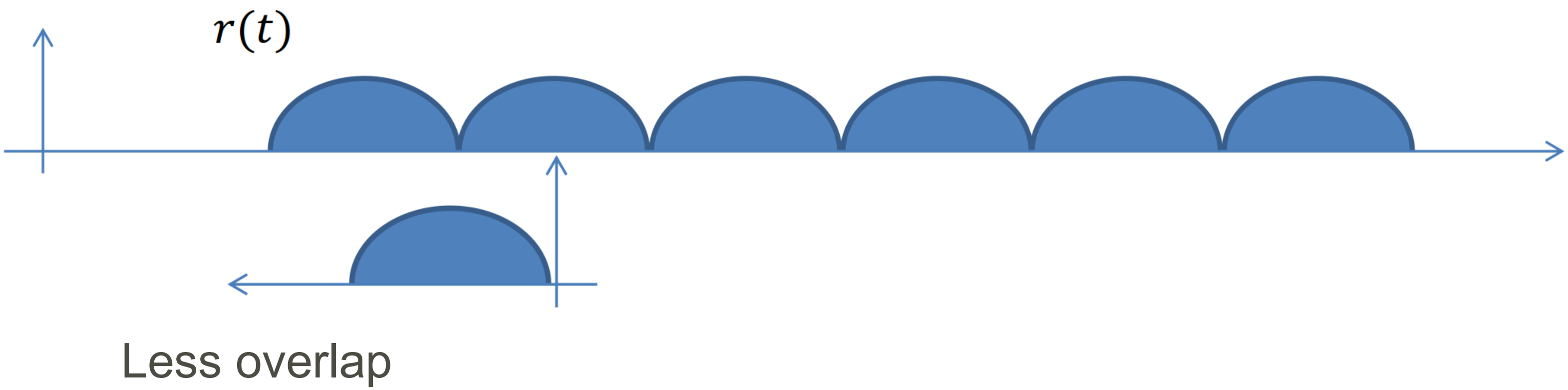

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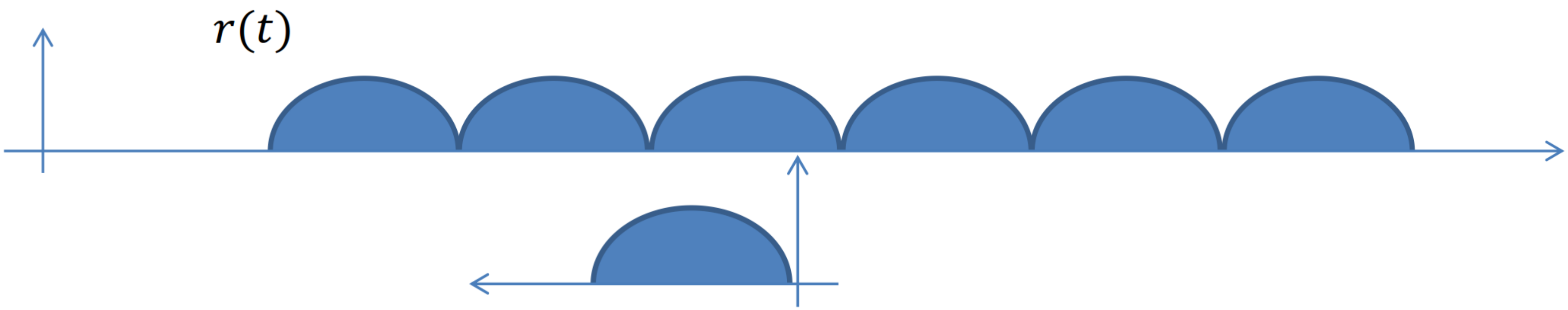

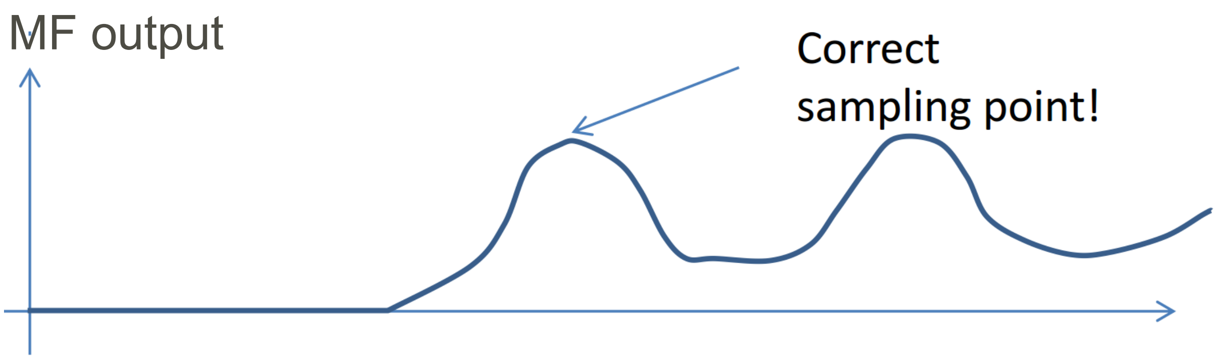

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