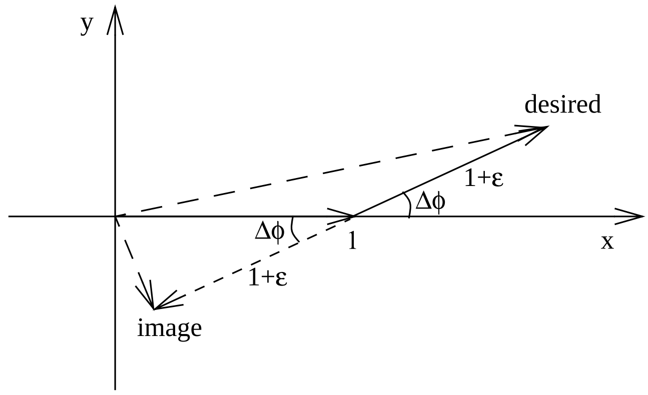
Solutions exercise 1 (Architectures)

1. Problem 18.3 (also problem 2a exam 01-03-06):



$$IRR = \frac{(\text{x-composant desired})^2 + (\text{y-composant desired})^2}{(\text{x-composant image})^2 + (\text{y-composant image})^2}$$

$$= \frac{[1 + (1+\varepsilon)\cos\Delta\phi]^2 + [(1+\varepsilon)\sin\Delta\phi]^2}{[1 - (1+\varepsilon)\cos\Delta\phi]^2 + [(1+\varepsilon)\sin\Delta\phi]^2} = \frac{1 + 2(1+\varepsilon)\cos\Delta\phi + (1+\varepsilon)^2}{1 - 2(1+\varepsilon)\cos\Delta\phi + (1+\varepsilon)^2}$$

The numerator is approximately equal to 4 for small ε and Δϕ.
The denominator N(Δϕ,ε) is expanded in a Taylor series around (Δϕ,ε)=(0,0):

$$\frac{\partial N}{\partial \Delta\phi} = 2(1+\varepsilon)\sin\Delta\phi = 0,\ \Delta\phi = \varepsilon = 0$$

$$\frac{\partial N}{\partial \varepsilon} = -2\cos\Delta\phi + 2 + 2\varepsilon = 0,\ \Delta\phi = \varepsilon = 0$$

$$\frac{\partial^2 N}{\partial \Delta\phi \partial \varepsilon} = 2\sin\Delta\phi = 0,\ \Delta\phi = \varepsilon = 0 \qquad \Rightarrow \qquad N \approx (\Delta\phi)^2 + \varepsilon^2$$

for small ε and Δϕ

$$\frac{\partial^2 N}{\partial (\Delta\phi)^2} = 2(1+\varepsilon)\cos\Delta\phi = 2,\ \Delta\phi = \varepsilon = 0$$

$$\frac{\partial^2 N}{\partial \varepsilon^2} = 2$$

That is

$$IRR \approx \frac{4}{(\Delta\phi)^2 + \varepsilon^2} \qquad \text{for small } \varepsilon \text{ and } \Delta\phi$$

Problem 2 (also problem 2b exam 01-03-06)

Bluetooth:

$$IRR = 20dB = 100 = \frac{4}{(\Delta\phi)^2 + \varepsilon^2} \Rightarrow (\Delta\phi)^2 + \varepsilon^2 = \frac{1}{25} \Rightarrow \Delta\phi \text{ or } \varepsilon = \frac{1}{5}$$

$$\Delta\phi = \frac{1}{5}\text{rad} = \frac{1}{5} \cdot \frac{180}{\pi} = 11° \qquad \text{or} \qquad \varepsilon = \frac{1}{5} = 20\%$$

This matching accuracy can easily be achieved in an integrated circuit.

Mobile phone:

$$IRR = 80dB = 10^8 = \frac{4}{(\Delta\phi)^2 + \varepsilon^2} \Rightarrow (\Delta\phi)^2 + \varepsilon^2 = \frac{4}{10^8} \Rightarrow \Delta\phi \text{ or } \varepsilon = 2 \cdot 10^{-4}$$

$$\Delta\phi = 0.011° \qquad \text{or} \qquad \varepsilon = \frac{1}{5} = 0.020\%$$

This is very difficult (or impossible) to achieve

3. Problem 18.6 with extension

Cubic nonlinearity: $v_o = a_0 + a_1 v_i + a_2 v_i^2 + a_3 v_i^3$

IIP_2: $|a_1 v_i| = |a_2 v_i^2| \Rightarrow |v_i| = \left|\frac{a_1}{a_2}\right| \Rightarrow IIP_2 = \frac{1}{R_{in}} \cdot \left(\frac{a_1}{a_2}\right)^2$

IIP_3: $|a_1 v_i| = |a_3 v_i^3| \Rightarrow v_i^2 = \left|\frac{a_1}{a_3}\right| \Rightarrow IIP_3 = \frac{1}{R_{in}} \cdot \left|\frac{a_1}{a_3}\right|$

$$\Rightarrow \quad \frac{IIP_2}{IIP_3} = \left|\frac{a_1^2/a_2^2}{a_1/a_3}\right| = \left|\frac{a_1 a_3}{a_2^2}\right|$$

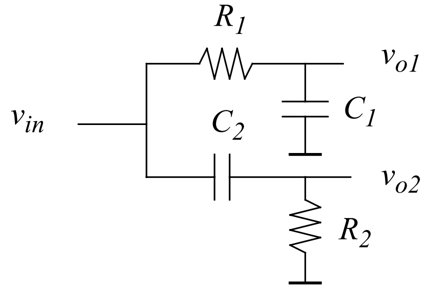
Extension. Assume $a_1 > 0$, $a_2 = 0$, $a_3 < 0$

ICP (1dB):

$$a_1 v_i - |a_3| v_i^3 = a_1 v_i \cdot 10^{-\frac{1}{20}} \Rightarrow a_1 v_i\left(1 - 10^{-\frac{1}{20}}\right) = |a_3| v_i^3 \Rightarrow ICP = \frac{1}{R_{in}} \cdot \left|\frac{a_1}{a_3}\right| \cdot \left(1 - 10^{-\frac{1}{20}}\right)$$

$$\Rightarrow \quad \frac{IIP_3}{ICP} = \frac{1}{\left(1 - 10^{-\frac{1}{20}}\right)} = 9.2 = 9.6\text{dB}$$

4. Phase difference versus frequency of RC-CR link



$$v_{o1} = v_{in}\frac{1/(j\omega C_1)}{R_1 + 1/(j\omega C_1)} = v_{in}\frac{1}{1 + j\omega R_1 C_1}$$

$$v_{o2} = v_{in}\frac{R_2}{R_2 + 1/j\omega C_2} = v_{in}\frac{j\omega R_2 C_2}{1 + j\omega R_2 C_2}$$

$$\frac{v_{o1}}{v_{o2}} = \frac{1}{j\omega R_2 C_2}\cdot\frac{1 + j\omega R_2 C_2}{1 + j\omega R_1 C_1} = \{R_1 C_1 = R_2 C_2 = RC\} = \frac{1}{j\omega RC}$$

If the time constants are selected equal, the phase difference is 90 degrees at all frequencies!

5. Design of RC-CR (problem 1 exam 010306 with extension)

See figure above. $R_1 = R_2 = 500\Omega$. Equal time constant give equal capacitors as well. To get equal amplitudes $\omega_0 RC$ must be equal to unity:

$$\omega_0 RC = 1 \Rightarrow C = \frac{1}{\omega_0 R} = \frac{1}{2\pi \cdot 10^9 \cdot 500} = 0.32\text{pF}$$

$$\left|\frac{v_{o2}}{v_{01}}\right| = \omega RC = \frac{\omega}{\omega_0} = \frac{f}{f_0} \qquad 0.95 < \frac{f}{f_0} < 1.05 \Rightarrow 950\text{MHz} < f < 1.05\text{GHz}$$

unloaded at ω_0: $$Z_{in} = \frac{1}{2}\left(R + \frac{1}{j\omega_0 C}\right) = \frac{1}{2}\left(R + \frac{RC}{jC}\right) = \frac{1}{2}(R - jR) = 250(1-j)\ \Omega$$

What happens when the ouputs are loaded by 5kΩ each?

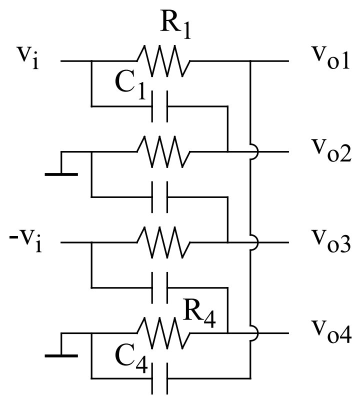
$$v_{o1} = v_{in}\frac{\dfrac{10R\cdot 1/sC}{10R + 1/sC}}{R + \dfrac{10R\cdot 1/sC}{10R + 1/sC}} = v_{in}\frac{10R\cdot\dfrac{1}{sC}}{\left(10R + \dfrac{1}{sC}\right)R + 10R\cdot\dfrac{1}{sC}} = v_{in}\frac{10R}{(1+10sRC)R + 10R}$$

$$= v_{in}\frac{10}{1 + 10sRC + 10} = v_{in}\frac{1}{\dfrac{11}{10} + sRC}$$

$$v_{o2} = v_{in}\frac{\dfrac{10}{11}R}{\dfrac{10}{11}R + \dfrac{1}{sC}} = v_{in}\frac{\dfrac{10}{11}sRC}{\dfrac{10}{11}sRC + 1} = v_{in}\frac{sRC}{\dfrac{11}{10} + sRC} \qquad \frac{v_{o2}}{v_{o1}} = sRC = j\omega RC$$

The outputs are still in quadrature and have equal amplitudes. The only difference is that the amplitude is reduced slightly (5%) by the loading.

6,7 Absolute and relative mismatch in a single stage polyphase filter



$$v_{o1} = v_i \cdot \frac{1/sC_4}{1/sC_4 + R_1} = v_i \cdot \frac{1}{1 + sR_1C_4}$$

$$v_{o2} = v_i \cdot \frac{R_2}{1/sC_1 + R_2} = v_i \cdot \frac{sR_2C_1}{1 + sR_2C_1}$$

$$v_{o3} = -v_i \cdot \frac{1/sC_2}{1/sC_2 + R_3} = -v_i \cdot \frac{1}{1 + sR_3C_2}$$

$$v_{o4} = -v_i \cdot \frac{R_4}{1/sC_3 + R_4} = -v_i \cdot \frac{sR_4C_3}{1 + sR_4C_3}$$

6. Absolute error:

No phase error will occur if all resistors and capacitors are equal (perfectly matched).

20% absolute error =>

$$\text{amplitude error} = \left|\frac{v_{o2}}{v_{o1}}\right| - 1 = \omega_0 RC - 1 = \left\{ RC = \frac{1}{\omega_o} \pm 20\% \right\} = \pm 20\%$$

7. Mismatch:

If **one** capacitor is 0.2% off, just **one** output signal will be affected.

$$\text{phase error} = \angle(1 + j\omega_0 RC_{nom}) - \angle(1 + j\omega_0 RC) = \left\{ \omega_0 = \frac{1}{RC_{nom}} \right\}$$

$$= \angle(1 + j) - \angle(1 + (j \pm 0.2\%)) = 0.06°$$