Solutions exercise 4 (Noise)

1. Problem 10.6

a.

$$R_{opt} = \frac{v_n}{i_n}$$

Amplifier A: $R_{opt} = \frac{10\text{nV}}{50\text{fA}} = 200\text{k}\Omega$ Amplifier B: $R_{opt} = \frac{10\text{nV}}{100\text{fA}} = 100\text{k}\Omega$

b. Use amplifier A and transform to 200kΩ. (Also without transformer, A gives less noise)

c.

$$\left.\begin{aligned} F_{min} &= 1 + 2 \cdot \frac{R_n}{R_{opt}} \\ R_n &= \frac{\overline{v_n^2}}{4\text{kT}\Delta f} = \frac{(10 \cdot 10^{-9})^2}{4 \cdot 4.00 \cdot 10^{-21}} = 6.25\text{k}\Omega \end{aligned}\right\} \Rightarrow F_{min} = 1 + 2 \cdot \frac{6.25}{200} = 1.065 = 0.3\text{dB}$$

2. Problem 10.9

Given: $A_v = 1000$ $BW = 1\text{kHz}$, single pole

a. $$\overline{v_{n,\,out}^2} = \overline{v_{ni}^2} \cdot A_v^2 \cdot BW \cdot \frac{\pi}{2} = 10^{-15} \cdot 10^6 \cdot 10^3 \cdot \frac{\pi}{2} = 1.57 \cdot 10^{-6}\ \text{V}^2$$

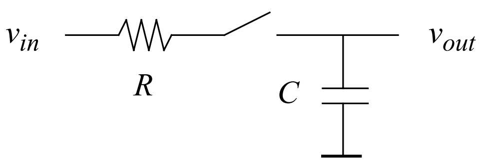
b. $\overline{v_{ni}^2} = 10^{-15} \cdot \left(1 + \frac{10\text{Hz}}{f}\right) \cdot \Delta f$ The same noise as in (a) plus additional 1/f-noise

$$\overline{v_{n,\,out,\,1/f}^2} \approx \int_{f_0}^{1000} 10^{-14} \cdot \frac{1}{f} \mathrm{d}f \cdot A_v^2 = 10^{-14} \cdot 10^6 \cdot \ln\left(\frac{1000}{f_0}\right)$$

$$f_0 = 1\text{mHz} \Rightarrow \overline{v_{n,\,out,\,1/f}^2} \approx 10^{-8} \cdot \ln(10^6) = 1.4 \cdot 10^{-7}\ \text{V}^2 \Rightarrow$$

$$\overline{v_{n,\,out}^2} = 1.57 \cdot 10^{-6} + 1.4 \cdot 10^{-7} = 1.7 \cdot 10^{-6}\ \text{V}^2$$

3. Problem 10.11



$$\overline{v_{nR}^2} = 4\text{kT}R \cdot \Delta f \qquad BW = \frac{1}{2\pi RC}$$

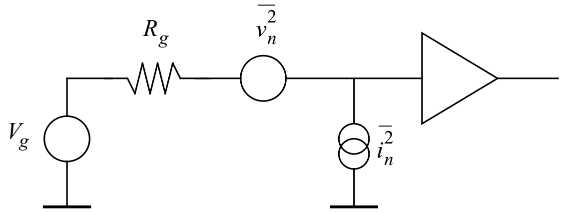
Single pole => Noise $BW = \frac{\pi}{2} \cdot BW = \frac{1}{4RC}$

$$\text{Total noise} = \overline{v_{nR}^2} \cdot \text{Noise } BW = 4\text{kT}R \cdot \frac{1}{4RC} = \frac{\text{kT}}{C}$$

The total noise only depends on C (independent of R). This is an important result for switched capacitor circuits.

4. Problem 10.12

First study the single-ended case:

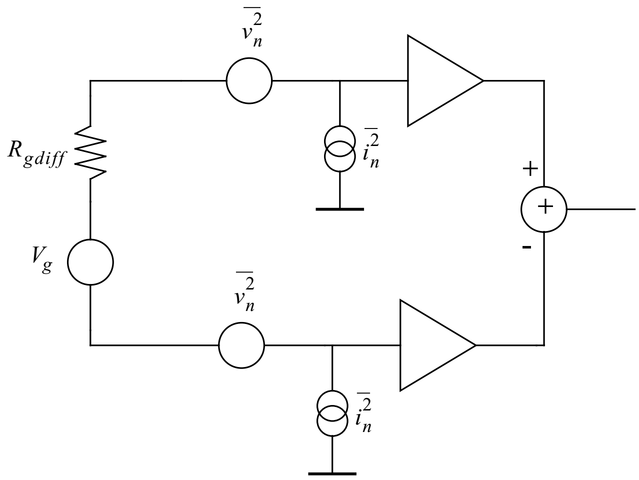


$$F = \frac{\overline{i_n^2} + \overline{v_n^2} \cdot \frac{1}{R_g^2} + 4\mathrm{kT} \cdot \frac{1}{R_g}}{4\mathrm{kT} \cdot \frac{1}{R_g}} = 1 + \frac{\overline{i_n^2} R_g^2 + \overline{v_n^2}}{4\mathrm{kT} R_g}$$

$$\frac{\partial F}{\partial R_g} = \frac{\overline{i_n^2}}{4\mathrm{kT}} - \frac{\overline{v_n^2}}{4\mathrm{kT}} \cdot \frac{1}{R_g^2} = 0 \Rightarrow R_{g,\,opt} = \sqrt{\overline{v_n^2}/\overline{i_n^2}}$$

$$F_{min} = 1 + \frac{\overline{i_n^2} R_{g,\,opt}^2 + \overline{v_n^2}}{4\mathrm{kT} R_{g,\,opt}} = 1 + \frac{\overline{i_n^2}(\overline{v_n^2}/\overline{i_n^2}) + \overline{v_n^2}}{4\mathrm{kT}\sqrt{\overline{v_n^2}/\overline{i_n^2}}} = 1 + \frac{2 \cdot \overline{v_n^2}}{4\mathrm{kT}} \cdot \sqrt{\overline{i_n^2}/\overline{v_n^2}} = 1 + \frac{2\sqrt{\overline{v_n^2} \cdot \overline{i_n^2}}}{4\mathrm{kT}}$$

Now move on to the differential one:



$$F = \frac{2 \cdot \overline{i_n^2} + 2 \cdot 4 \cdot \overline{v_n^2}/R_{gdiff}^2 + 4 \cdot 4\mathrm{kT}/R_{gdiff}}{4 \cdot 4\mathrm{kT}/R_{gdiff}} = \frac{2 \cdot \overline{i_n^2} \cdot R_{gdiff}^2 + 8 \cdot \overline{v_n^2} + 16\mathrm{kT} \cdot R_{gdiff}}{16\mathrm{kT} \cdot R_{gdiff}}$$

$$= \frac{\overline{i_n^2} \cdot R_{gdiff}^2 + 4 \cdot \overline{v_n^2} + 8\mathrm{kT} \cdot R_{gdiff}}{8\mathrm{kT} \cdot R_{gdiff}} = 1 + \frac{\overline{i_n^2} \cdot R_{gdiff}^2 + 4 \cdot \overline{v_n^2}}{8\mathrm{kT} \cdot R_{gdiff}}$$

$$\frac{\partial F}{\partial R_{gdiff}} = \frac{\overline{i_n^2}}{8\mathrm{kT}} - \frac{\overline{v_n^2}}{2\mathrm{kT} \cdot R_{gdiff}^2} = 0 \Rightarrow R_{gdiff,\,opt} = 2 \cdot \sqrt{\overline{v_n^2}/\overline{i_n^2}} = 2 \cdot R_{g,\,\mathrm{se},\,opt}$$

$$F_{min} = 1 + \frac{\overline{i_n^2} \cdot R_{gdiff,\,opt}^2 + 4 \cdot \overline{v_n^2}}{8\mathrm{kT} \cdot R_{gdiff,\,opt}} = \ldots = 1 + \frac{2\sqrt{\overline{v_n^2} \cdot \overline{i_n^2}}}{4\mathrm{kT}} = F_{min,\,se}$$

Result: The optimum differential source resistance is twice the optimum single ended resistance, and the minimum **noise figure is the same in both cases**.