Solutions exercise 8 (PA & Bandgap)

1. Problem 3 from exam 01-03-06

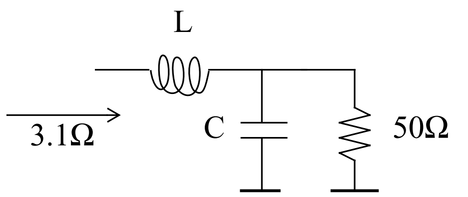
a. **GSM:**

$R_L = 50\Omega \qquad P_{out} = 30\text{dBm} = 1\text{W}_{\text{rms}} \qquad f = 900\text{MHz}$

$V_L = \sqrt{2P_{out}R_L} = \sqrt{100} = 10\text{V}_{\text{pk}} = 20\text{V}_{\text{pp}}$

$V_{PA} = 5\text{V}_{\text{pp}} \Rightarrow \text{Transform } \frac{20}{5} = 4 \text{ times in voltage } = 16 \text{ times in impedance}$

$R_{PA} = \frac{50\Omega}{16} = 3.1\Omega$



$Q = \text{voltage transformation} = 4$

$BW = \frac{f}{Q} = 220\text{MHz}$

$L = \frac{QR_{PA}}{\omega_0} = 2.2\text{nH} \qquad C = \frac{Q}{\omega_0 R_L}$

$\hat{I}_L = \frac{\hat{V}_L}{R_L} = \frac{10}{50} = 0.2\text{A} \qquad \hat{I}_{PA} = \hat{I}_L \cdot Q = 0.8\text{A} = \hat{I}_{ind}$

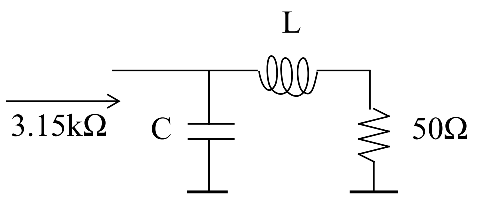
Bluetooth:

$R_L = 50\Omega \qquad P_{out} = 0\text{dBm} = 1\text{mW}_{\text{rms}} \qquad f = 2.4\text{GHz}$

$V_L = \sqrt{2P_{out}R_L} = \sqrt{0.1} = 0.32\text{V}_{\text{pk}} = 0.63\text{V}_{\text{pp}}$

$V_{PA} = 5\text{V}_{\text{pp}} \Rightarrow \text{Transform } \frac{5}{0.63} = 7.9 \text{ times in voltage } = 63 \text{ times in impedance}$

$R_{PA} = 50\Omega \cdot 63 = 3.15\text{k}\Omega$



$Q = \text{voltage transformation} = 7.9$

$BW = \frac{f}{Q} = 300\text{MHz}$

$L = \frac{QR_L}{\omega_0} = 26\text{nH} \qquad C = \frac{1}{\omega_0^2 L} = 170\text{fF}$

$\hat{I}_L = \hat{I}_{ind} = \sqrt{2PR_L} = \sqrt{\frac{0.002}{50}} = 6.3\text{mA}_{\text{pk}}$

b. **GSM:**

$\hat{I}_{PA} = 0.8\text{A} = I_{dc}$ for class A operation

smallest v_{ds} at 5V$_{pp}$ and $V_{dd} = 3\text{V}$ is equal to 0.5V $\left(V_{dd} - \frac{5V_{pp}}{2} = 3 - 2.5 = 0.5\text{V}\right)$

At this instant the current is equal to 2 times $I_{dc} = 1.6\text{A}$

To avoid the triode region V_{od} must be less than $V_{ds} = 0.5\text{V}$

That is, at 0.5V overdrive the transistor must be able to sink 1.6A:

$$I_d = \frac{1}{2}\mu C_{ox}\frac{W}{L}V_{od}^2 \Rightarrow W = \frac{2I_dL}{\mu C_{ox}V_{od}^2} = \frac{2 \cdot 1.6 \cdot 0.4 \cdot 10^{-6}}{110 \cdot 10^{-6} \cdot 0.5^2} = 46\text{mm}$$

(This is a very large transistor!)

Bluetooth:

$$\hat{I}_{PA} = \frac{2.5V_{pk}}{3.15\text{k}\Omega} = 0.8\text{mA} = I_{dc}$$

$$W = \frac{2I_dL}{\mu C_{ox}V_{od}^2} = \frac{2 \cdot 1.6 \cdot 10^{-3} \cdot 0.4 \cdot 10^{-6}}{110 \cdot 10^{-6} \cdot 0.5^2} = 46\mu\text{m}$$

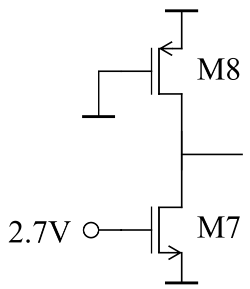
(The width is 1000 times smaller, just like the output power)

2. Problem 9.11

a. If the circuit has not started, the gate of M9 is at vdd potential. M9 then conducts heavily and force current through M3 and Q1, making the circuit start. When the circuit starts current flow through M5, M6, M7 so that the gate potential of M9 is reduced and M9 turns off.

b.

$V_{gs9} = V_T - 0.4\text{V} \Rightarrow \frac{(W/L)_7}{(W/L)_8} = ?$ Assume $V_{dd} = 3\text{V}$ and $V_{G7max} = 2.7\text{V}$



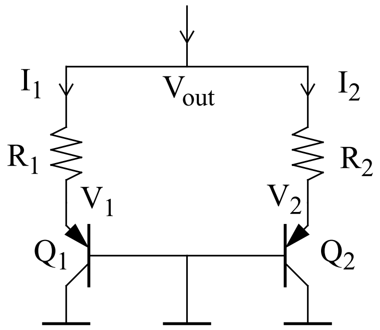
$$V_{G9} = V_{BE1} + V_T - 0.4\text{V} = 0.713 + 0.6 - 0.4 = 0.913\text{V}$$

M7 and M8 operate in the triode region with the same drain current:

$$\mu_n C_{ox} \cdot \left(\frac{W}{L}\right)_7 \cdot \left[(V_{G7} - V_{Tn}) \cdot V_{G9} - \frac{V_{G9}^2}{2}\right] = \mu_p C_{ox} \cdot \left(\frac{W}{L}\right)_8 \cdot \left[(V_{dd} - V_{Tp}) \cdot (V_{dd} - V_{G9}) - \frac{(V_{dd} - V_{G9})^2}{2}\right]$$

$$\frac{(W/L)_7}{(W/L)_8} = \frac{\mu_p}{\mu_n} \cdot \frac{(V_{dd} - V_{Tp}) \cdot (V_{dd} - V_{G9}) - (V_{dd} - V_{G9})^2/2}{(V_{G7} - V_{Tn}) \cdot V_{G9} - V_{G9}^2/2} = 0.6$$

Problem 9.4



$$I_1 = (V_{out} - V_1)\cdot\frac{1}{R_1} = \left(V_{out} - V_{GO} + V_T\cdot\ln\left(\frac{A_E B T^r}{I_{c1}}\right)\right)\cdot\frac{1}{R_1}$$

$$= \left(V_{out} - V_{GO} + V_T\cdot\ln\left(\frac{A_E B T^r}{I_1\cdot\frac{\beta_1}{\beta_1+1}}\right)\right)\cdot\frac{1}{R_1}$$

$$= \left(V_{out} - V_{GO} + V_T\cdot\ln\left(\frac{A_E B T^r}{I_1}\right) + V_T\cdot\ln\left(1+\frac{1}{\beta_1}\right)\right)\cdot\frac{1}{R_1}$$

$$I_2 = \left(V_{out} - V_{GO} + V_T\cdot\ln\left(\frac{mA_E B T^r}{I_2\cdot\frac{\beta_2}{\beta_2+1}}\right)\right)\cdot\frac{1}{R_2}$$

$$= \left(V_{out} - V_{GO} + V_T\cdot\ln\left(\frac{mA_E B T^r}{I_2}\right) + V_T\cdot\ln\left(1+\frac{1}{\beta_2}\right)\right)\cdot\frac{1}{R_2}$$

I_{ideal} and $V_{out,ideal}$ is the current and voltage when β is infinite

$$I_1 = I_{ideal} + V_T\cdot\ln\left(1+\frac{1}{\beta_1}\right)\cdot\frac{1}{R_1} + (V_{out} - V_{ideal})\cdot\frac{1}{R_1}$$

$$I_2 = I_{ideal} + V_T\cdot\ln\left(1+\frac{1}{\beta_2}\right)\cdot\frac{1}{R_2} + (V_{out} - V_{ideal})\cdot\frac{1}{R_2}$$

$$I_1 = I_2 \Rightarrow \frac{V_T\cdot\ln\left(1+\frac{1}{\beta_1}\right) + (V_{out} - V_{ideal})}{R_1} = \frac{V_T\cdot\ln\left(1+\frac{1}{\beta_2}\right) + (V_{out} - V_{ideal})}{R_2}$$

$$\beta_1 = \beta_2 = \beta \Rightarrow V_{err} = V_{out} - V_{ideal} = -V_T\cdot\ln\left(1+\frac{1}{\beta}\right)$$

Example: $\beta = 10 \Rightarrow V_{err} = -25\text{mV}\cdot\ln 1.1 = -2.4\text{mV}$