PHD COURSE in Operations Research (November – December 2008)

Integer programming with applications to engineering problems

load:	**lectures 28 h, exercises 14 h, laboratory 20 h, self-study 62 h**	
ECTS credits:	**12**	**(8 old points)**
duration:	**7 weeks**	**(effective time of classes: 3 × 3 days)**
language:	**English**	
coordinator:	**prof. Michal Pióro**	**(mpp@eit.lth.se)**

Goals: The course will provide the PhD students with knowledge of (mixed) integer programming (MIP) – the basic optimization approach to real-life engineering problems, and will illustrate the main methods of MIP on a set of representative problems. Besides, the students will get familiar with AMPL/CPLEX – one of the best optimization packages for solving MIP problems.

General description:

Integer programming (more precisely, mixed-integer programming – MIP) is a basic approach to real-life engineering problems, including communication and electrical network design, transportation routing, crew scheduling, etc. Such real-life problems can be modelled using MIP formulations and tend to be NP-hard. So in order to seek for effective solutions one has usually to try the whole spectrum of MIP methods, ranging from linear programming based branch-and-bound approach to stochastic heuristic methods, frequently applying some kind of decomposition. MIP modelling gives an excellent opportunity for understanding what the real-life optimization problems are and how they can be approached. During the course we will develop the basic knowledge about linear programming (the simplex method), cutting plane method, branch-and-bound, valid inequalities and branch-and-cut, Dantzig-Wolfe decomposition and column generation, and stochastic heuristics. We will show how to apply the IP methods to multi-commodity flow network design, vehicle routing, manpower planning, job scheduling, and plant location.

Lectures (2 hour units):

1. Introduction. An illustrative example: multi-commodity flow network design problem. Simple and difficult variants of the problem.
2. Linear programming I. Basic notions and properties of LP problems. Basic features of the simplex method.
3. Modelling non-linearity. Convex and concave problems and the crucial differences between the two. Applications to network design (Frank-Wolfe algorithm and Yaged algorithm).
4. Branch-and-bound (B&B) method and algorithm for problems involving binary variables. Extension to a general MIP formulation. Potential effectiveness of B&B.
5. Application of B&B to a fixed-charge network design problem and to a single-path allocation problem.
6. Stochastic heuristics. Simulated annealing (application to travelling salesman problem) and evolutionary algorithm (application to single-path routing).
7. General form of an optimization problem. Relaxation, linear relaxation, Lagrangean relaxation (LR). Elements of the dual theory. Duality in convex programming. Duality gap.

8. Duality in linear programming and MIPs. The difference between linear relaxation and LR. Column generation and Dantzig-Wolfe decomposition principle. Application: single-path allocation problem.
9. Cutting plane method, valid inequalities and branch-and-cut (B&C) method. Branch-and-price (B&P) method.
10. The notion of NP-completeness. Pseudo-polynomial problems. Separation theorem.
11. Applications: resilient design of multi-commodity flow networks I.
12. Applications: resilient design of multi-commodity flow networks II.
13. Applications: vehicle routing and manpower planning.
14. Applications: job scheduling.and plant location.

Exercises: The lectures will be illustrated by exercises consisting in solving example problems (also by hand).

Laboratory: The students will be instructed how to apply the AMPL/CPLEX optimization suite for solving MIP problems, and will solve a set of illustrative example problems.

Self-study: The role of self-study will be two-fold. First, the students will study the theory behind MIP and solve appropriate, individually assigned exercises illustrating the theory. Second, the students will prepare the main individual project assignment to be solved using AMPL and CPLEX, and write a report.

Literature:

1. M. Pióro and D.Medhi. *Routing, Flow, and Capacity Design of Communication and Computer Networks*. Morgan-Kaufmann, 2004.
2. M.Minoux. *Mathematical Programming - Theory and Algorithms*. Wiley, 1995.
3. H.P.Williams, *Model Building in Mathematical Programming*. Wiley, 1993.
4. L.A.Wolsey. *Integer Programming*. Wiley, 1998.

Schedule:

The course will consist of theree sessions held at LTH, and of individual assignments. Each of the three sessions (called weeks below) will be organized during four consecutive days. The lecture unit is equal to 45 minuts, followed by a 15 minutes break. The detailed plan:

Week 1 (week 47)

Day 1:	13:00 – 16:00	lecture	(3 units)
Day 1:	16:00 – 17:00	exercises	(1 unit)
Day 2:	09:00 – 12:00	lecture	(3 units)
Day 2:	13:00 – 15:00	exercises	(2 units)
Day 2:	15:00 – 17:00	lecture	(2 units)
Day 3:	09:00 – 12:00	lecture	(3 units)
Day 3:	13:00 – 15:00	exercices	(2 units)
Day 3:	15:00 – 17:00	laboratory	(2 units)
Day 4:	09:00 – 12:00	laboratory	(3 units)

Week 2 (week 49)

Day 1:	13:00 – 16:00	lecture	(3 units)
Day 1:	16:00 – 17:00	exercises	(1 unit)
Day 2:	09:00 – 12:00	lecture	(3 units)
Day 2:	13:00 – 15:00	exercises	(2 units)
Day 2:	15:00 – 17:00	lecture	(2 units)
Day 3:	09:00 – 12:00	lecture	(3 units)
Day 3:	13:00 – 15:00	exercices	(2 units)
Day 3:	15:00 – 17:00	laboratory	(2 units)
Day 4:	09:00 – 12:00	laboratory	(3 units)

Week 3 (week 51)

Day 1:	13:00 – 15:00	lecture	(2 units)
Day 1:	15:00 – 17:00	exercises	(2 units)
Day 2:	09:00 – 11:00	lecture	(2 units)
Day 2:	11:00 – 12:00	exercises	(1 unit)
Day 2:	13:00 – 17:00	laboratory	(4 units)
Day 3:	09:00 – 11:00	lecture	(2 units)
Day 3:	11:00 – 12:00	exercices	(1 unit)
Day 3:	13:00 – 17:00	laboratory	(4 units)
Day 4:	09:00 – 11:00	laboratory	(2 units)
Day 4:	11:00 – 12:00	test in writing	(1 units)

Remark 1: Please note that the participants from outside LTH will be able to come for the classes to Lund for only 3 nights during each session week.

Remark 2: The partcicular weeks (weeks 47, 49, 51) can be changed if the partcipants will consider this advantageous.